Physics Letters B 704 (2011) 102–107

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Early dark energy from zero-point quantum fluctuations

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ARTICLE INFO

Article history:
Received 14 June 2011
Received in revised form 24 July 2011
Accepted 4 September 2011
Available online 7 September 2011
Editor: L. Alvarez-Gaumé

Keywords:
Early dark energy
Cosmological constant
Vacuum fluctuations

ABSTRACT

We examine a cosmological model with a dark energy density of the form $\rho_{\rm DE}(t)=\rho_X(t)+\rho_Z(t)$, where ρ_X is the component that accelerates the Hubble expansion at late times and $\rho_Z(t)$ is an extra contribution proportional to $H^2(t)$. This form of $\rho_Z(t)$ follows from the recent proposal that the contribution of zero-point fluctuations of quantum fields to the total energy density should be computed by subtracting the Minkowski-space result from that computed in the FRW space–time. We discuss theoretical arguments that support this subtraction. By definition, this eliminates the quartic divergence in the vacuum energy density responsible for the cosmological constant problem. We show that the remaining quadratic divergence can be reabsorbed into a redefinition of Newton's constant only under the assumption that $\nabla^\mu\langle 0|T_{\mu\nu}|0\rangle=0$, i.e. that the energy–momentum tensor of vacuum fluctuations is conserved in isolation. However in the presence of an ultra-light scalar field X with $m_X<H_0$, as typical of some dark energy models, the gravity effective action depends both on the gravitational field and on the X field. In this case general covariance only requires $\nabla^\mu(T^X_{\mu\nu}+\langle 0|T_{\mu\nu}|0\rangle)$. If there is an exchange of energy between these two terms, there are potentially observable consequences. We construct an explicit model with an interaction between ρ_X and ρ_Z and we show that the total dark energy density $\rho_{\rm DE}(t)=\rho_X(t)+\rho_Z(t)$ always remains a finite fraction of the critical density at any time, providing a specific model of early dark energy. We discuss the implication of this result for the coincidence problem and we estimate the model parameters by means of a full likelihood analysis using current CMB, SNe Ia and BAO data.

1. Introduction

Understanding the origin of dark energy is one of the most important challenges facing cosmology and theoretical physics (see e.g. [1–4]). One aspect of the problem is to understand what is the role of zero-point vacuum fluctuations in cosmology. In a Friedmann–Robertson–Walker (FRW) metric with Hubble parameter $H(t)$ the bare vacuum energy density takes the form

$$\left[\rho_{\rm bare}(\Lambda_c)\right]_{\rm FRW}=\left[\rho_{\rm bare}(\Lambda_c)\right]_{\rm Mink}+\mathcal{O}\left(H^2(t)\Lambda_c^2\right), \tag{1}$$

where $[\rho_{\rm bare}(\Lambda_c)]_{\rm Mink}$ is the bare vacuum energy density in Minkowski space, whose leading divergence is $\mathcal{O}(\Lambda_c^4)$, and we used for definiteness a momentum space cutoff Λ_c. In the usual treatment this Λ_c^4 divergence is reabsorbed into a renormalization of the cosmological constant, giving rise to the cosmological constant problem. The divergence $\propto H^2\Lambda_c^2$ is instead absorbed into a renormalization of Newton's constant G [5,6].

In this Letter, expanding on results presented in [7], we reexamine the role of vacuum energies in cosmology. First, we will propose theoretical arguments suggesting that the correct way of computing the physical vacuum energy is to subtract the bare vacuum energy density of Minkowski space, $[\rho_{\rm bare}(\Lambda_c)]_{\rm Mink}$, from the FRW result given in Eq. (1), before renormalizing the result. By definition this subtraction eliminates the troublesome Λ_c^4 divergence and, therefore, the cosmological constant problem. Then we turn our attention to the left over term $H^2\Lambda_c^2$ which now becomes the leading term in the vacuum energy. It is usually believed that this quadratic divergence can be reabsorbed into a renormalization of G. We show that this is correct only under the assumption that vacuum expectation value (VEV) of the energy–momentum tensor is conserved in isolation (an assumption that was implicit in the literature). However, general covariance of General Relativity (GR) only implies the conservation of the *total* energy–momentum tensor $T_{\mu\nu}+\langle 0|T_{\mu\nu}|0\rangle$, including both the classical term $T_{\mu\nu}$ and the semiclassical term $\langle 0|T_{\mu\nu}|0\rangle$. The separate conservation of $\langle 0|T_{\mu\nu}|0\rangle$ only takes place if we can define an effective action which depends only on the gravitational field, by integrating out the matter degrees of freedom. This is possible only if the matter degrees of freedom are heavy with respect to the energy scale of the problem, and can then be integrated out. In a cosmological setting, this means that matter fields should satisfy $m>H_0$. If, in contrast, there is an ultra-light scalar fields with $m<H_0$, as is typical of some quintessence model, this field cannot be integrated out from the effective low energy action. We show that, as a

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.09.010

result, in general $\nabla^\mu T^X_{\mu\nu} = -\nabla^\mu \langle 0|T_{\mu\nu}|0\rangle \neq 0$. In this case the effect of the quadratically divergent term in the vacuum fluctuations cannot simply be absorbed into a renormalization of Newton's constant G, and gives rise to interesting and potentially detectable cosmological effects. We construct a specific coupled early dark energy model and test it against current observations.

We use natural units where $\hbar = c = 1$, $G = M_{\rm Pl}^{-2}$. If not specified otherwise, we work in a spatially flat FRW metric with signature $(-+++)$, cosmic time t, scale factor $a(t)$ and Hubble parameter $H(t) = (da/dt)/a$. Today, the Hubble parameter and the critical density take the values H_0 and $\rho_0 = 3H_0^2/(8\pi G)$, respectively.

2. Subtraction of the flat-space vacuum energy

In Minkowski space the divergence in the vacuum energy density is usually dealt with by normal ordering the Hamiltonian, which gives by definition a vanishing result for the physical vacuum energy density. However, it is useful to realize that the problem can be treated more generally in the context of renormalization theory, which rather allows us to fix the renormalized vacuum energy density to any observed value. In the standard language of renormalization, divergences in a generic N-point Green's function are cured by adding the corresponding counterterms to the Lagrangian density. The same procedure can be applied to vacuum energy, i.e. to the $N = 0$ Green's function: one simply adds a constant counterterm $-\rho_{\rm count}(\Lambda_c)$ to the Lagrangian density. This corresponds to adding a term $+\rho_{\rm count}(\Lambda_c)$ to the Hamiltonian density. Hence the renormalized, physical vacuum energy density is given by $\rho_{\rm ren} = \rho_{\rm bare}(\Lambda_c) + \rho_{\rm count}(\Lambda_c)$. As always in renormalization theory, the counterterm $\rho_{\rm count}$ is chosen so to cancel the divergences in $\rho_{\rm bare}$ and leave us with the desired finite part that is fixed by comparison with the experiment.

Using the language of renormalization theory is useful in this context because it makes clear that the cosmological constant problem is not that quantum field theory (QFT) gives a *wrong* prediction for the cosmological constant (as it is sometimes incorrectly said). Strictly speaking QFT makes no prediction for the cosmological constant, just as it does not predict the electron mass nor the fine structure constant. Rather, it is a problem of naturalness, in the sense that the counterterm $\rho_{\rm count}(\Lambda_c)$ must be fine-tuned to exceeding accuracy, in order to cancel the Λ_c^4 divergence in $\rho_{\rm bare}$, leaving a physical vacuum energy density that, if one identifies Λ_c with the Planck mass, is about $\mathcal{O}(10^{120})$ times smaller than Λ_c^4.

Posing the problem in terms of a cancellation between $\rho_{\rm bare}(\Lambda_c)$ and $\rho_{\rm count}(\Lambda_c)$ can also give a first hint for a possible solution. First of all, one should appreciate that neither the bare vacuum energy $\rho_{\rm bare}(\Lambda_c)$ nor the counterterm $\rho_{\rm count}(\Lambda_c)$ have a physical meaning and only their sum is an observable. Thus, this kind of cancellation is different from a fine-tuning between observable quantities. Indeed, the Casimir effect is a well-known example where a rather similar cancellation takes place. In that case the physical vacuum energy density of a quantum field in a finite volume is found by taking the difference between the bare vacuum energy density computed in this finite volume and the bare vacuum energy density in an infinite volume. Regularizing with a cutoff Λ_c both terms diverge as Λ_c^4, but their difference is finite and depends only on the physical size of the system. This might suggest that, similarly, to obtain the physical effect of the vacuum energy density in cosmology, one should compute the vacuum energy density in a FRW space–time and subtract from it the value computed in a reference geometry, which could be naturally taken as Minkowski space, leading to a sort of "cosmological Casimir effect".

Before taking this analogy with the Casimir effect seriously, one must however face the obvious objection that in special relativity the zero of the energy can be chosen arbitrarily, and only energy differences with respect to the ground state are relevant.[1] In contrast, in GR we cannot chose the zero of the energy arbitrarily. One typically expects that "every form of energy gravitates", so the contribution of Minkowski space cannot just be dropped.

While it is certainly true that in GR the choice of the zero for the energy is not arbitrary, the point that we wish to make here is that what is the correct choice can be a non-trivial issue. As a first example, consider the definition of energy for asymptotically flat space–times. This is obtained from the Hamiltonian formulation of GR, which goes back to the classic paper by Arnowitt, Deser and Misner (ADM) [8,9]. To properly define the Hamiltonian of a given field configuration in GR one must work at first in a finite three-dimensional volume V. Then the Hamiltonian takes the form $H_{\rm GR} = H_{\rm bulk} + H_{\rm boundary}$, where $H_{\rm bulk}$ is given by an integral over the spatial volume V at fixed time, while $H_{\rm boundary}$ is given by an integral over the two-dimensional boundary ∂V. When one evaluates $H_{\rm bulk}$ on any classical solution of the equations of motion one finds a vanishing result (since $H_{\rm bulk}$ is proportional to the constraint equations of GR), so the whole contribution comes from the boundary term. On the other hand, $H_{\rm boundary}$ diverges for any asymptotically flat metric $g_{\mu\nu}$ (including flat space–time), when the boundary is finally taken to infinity. The solution proposed by ADM is to subtract from this boundary term the same term computed in Minkowski space $\eta_{\mu\nu}$. Accordingly, the energy E associated with a classical asymptotically flat metric $g_{\mu\nu}$ is obtained by defining

$$E = H_{\rm GR}[g_{\mu\nu}] - H_{\rm GR}[\eta_{\mu\nu}]. \tag{2}$$

This provides the standard definition of mass in GR, and reproduces the expected properties of asymptotically flat space–times. For instance, when applied to the Schwarzschild space–time, it correctly gives the mass that appears in the Schwarzschild metric. This underlines that our intuition that any form of energy gravitates according to GR is not entirely correct: Eq. (2) tells us that the energy associated to Minkowski space does not gravitate.

Similar subtractions also hold for non-asymptotically flat space–times, and can be performed either by subtracting the contribution of some reference space–time whose boundary has the same induced metric as the background under consideration [10–13], or even without introducing a reference background but just by adding some local counterterms to the boundary action, given by a coordinate-invariant functional of the intrinsic boundary geometry [14,15]. The latter prescription is particularly appealing for asymptotically AdS space–times. In fact, in the context of the AdS/CFT correspondence, this way of removing divergences in the gravitational action on the AdS side corresponds to the renormalization of the UV divergences in the conformal QFT that lives on the boundary [14–18].

These examples show that, already in classical GR, the energy that actually acts as a source of gravity can be obtained from a Hamiltonian only after performing an appropriate subtraction. It is quite natural to assume that the same should hold at the quantum level, so in particular for zero-point fluctuations of quantum fields in curved space. To understand what is the appropriate subtraction for the FRW metric, we consider the Friedmann equation that results from the Einstein equations sourced by $T^\mu_\nu + \langle 0|T^\mu_\nu|0\rangle$,

[1] Equivalently, one may observe that in the Casimir effect one actually measures the force between the plates, i.e. not the energy density itself but only its derivative w.r.t. the size of the system L. Since the divergence Λ_c^4 is independent of L, it can simply be dropped.