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Constraining decaying dark matter

Ran Huo

Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, United States

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ABSTRACT

We revisited the decaying dark matter (DDM) model, in which one collisionless particle decays early into two collisionless particles, that are potentially dark matter particles today. The effect of DDM will be manifested in the cosmic microwave background (CMB) and structure formation. With a systematic modification of CMB calculation tool `camb`, we can numerically calculate this effect, and compare it to observations. Further Markov Chain Monte Carlo `cosmomc` runnings update the constraints in that model: the free streaming length $\lambda_{FS} \lesssim 0.5$ Mpc for nonrelativistic decay, and $(\frac{M_{\rm DDM}}{\rm keV} Y)^2 \frac{T_d}{\rm yr} \lesssim 5\times 10^{-5}$ for relativistic decay.

1. Introduction

Decaying dark matter (DDM) in which the decay process happens at an early stage of the universe is natural for many models, and it is sometimes introduced as a way to adjust the DM relic density, because after decay the DM relic density will naively be lowered by

$$\Omega_d = \frac{\sum m_{\rm product}}{m_{\rm original}} \Omega_{\rm DDM}. \tag{1}$$

One example is in [1], in which gravitino, overproduced by reheating after inflation, will decay into the true lightest supersymmetric particles (LSP) axino as well as an axion.

However, pure gravitational constraints for that decay process are less understood and sometimes even simply ignored. Actually the model in [1] with their parameters should be ruled out [2]. Here we will present a model independent computation, in which the effect is calculated from the first principle, and can be compared directly with cosmological observation. Our model have both the parent particle and the daughter particles interacting very weakly, so the effect can only be manifested in gravitational effect, such as the cosmic microwave background (CMB) and the large scale structure formation. Our approach is based on a systematic modification of the CMB codes.

2. Principle

`cmbfast` [3] and `camb` [4] are CMB calculation tools which are based on the photon line of sight integration technique, instead of solving the Boltzmann equation explicitly. They work in the *synchronous gauge*, the metric perturbation of which is gauged completely into the spatial 3×3 part

$$g_{\mu\nu} = a^2 \begin{pmatrix} -1 & \\ & \delta_{ij} + h_{ij} \end{pmatrix}, \tag{2}$$

and the graviton h_{ij} can be decomposed into Fourier modes h and η

$$h_{ij}(\vec{x}) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} e^{i\vec{k}\cdot\vec{x}} \left(\hat{k}_i\hat{k}_j h(\vec{k}) + \left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right) 6\eta(\vec{k})\right). \tag{3}$$

Then linearized Einstein equation gives the equations of motion of h and η components [5]

$$k^2\eta - \frac{1}{2}\frac{\dot{a}}{a}\dot{h} = 4\pi G a^2 \delta T^0_0, \tag{4a}$$

$$k^2\dot{\eta} = 4\pi G a^2 (\bar{\rho} + \bar{p})\theta, \tag{4b}$$

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} - 2k^2\eta = -8\pi G a^2 \delta T^i_i, \tag{4c}$$

$$\ddot{h} + 6\ddot{\eta} + 2\frac{\dot{a}}{a}(\dot{h} + 6\dot{\eta}) - 2k^2\eta = -24\pi G a^2(\bar{\rho} + \bar{p})\sigma. \tag{4d}$$

Here overdot $\cdot$ means derivative to conformal time τ. θ is the peculiar velocity which is defined by $(\bar{\rho} + \bar{p})\theta \equiv i k^i \delta T^0_i$, and σ is the shear which is defined by $(\bar{\rho} + \bar{p})\sigma \equiv -(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})\Sigma^i_j = -(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})(T^i_j - \frac{1}{3}\delta^i_j T^k_k)$.

With the h and η metric perturbation, the Boltzmann equation in terms of the fractional perturbation Ψ is [5]

$$\frac{\partial\Psi}{\partial\tau} + i\frac{qk}{\epsilon}(\hat{k}\cdot\hat{n})\Psi + \frac{\partial \ln f(q)}{\partial \ln q}\left(\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2}(\hat{k}\cdot\hat{n})^2\right)$$

E-mail address: huor@uchicago.edu.

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$$= \frac{1}{f(q)}\left(\frac{\partial(f+\delta f)}{\partial\tau}\right)_C, \tag{5}$$

where $q = ap$ is the comoving momentum and is *conserved* in expansion if the particle is collisionless, $\epsilon = \sqrt{q^2 + a^2m^2}$ is the comoving energy, and $\hat{n}$ is the direction of the macroscopic flow of the fluid. $f(q)$ is the unperturbed partition function, and the real partition function can be defined with fractional perturbation $f(q,\tau,x^i,n_i) = f(q)(1+\Psi(q,\tau,x^i,n_i))$. Usually $f(q)$ is thermal distribution such as Fermi–Dirac distribution or Bose–Einstein distribution, but in our DDM model for daughter particles it is determined by the decay process.

After we plug into the collision term, we will find the formal photon line-of-sight integration solution to the Boltzmann equation. The anisotropy $\Delta_T \equiv \frac{\delta T}{T} \sim \frac{1}{4}\Psi_\gamma$ *today* is given by [3]

$$\Delta_T(\vec{k},\hat{n}) = \int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)} e^{\kappa}\Big[\left(\dot{\eta} - \alpha\mu^2k^2\right) + \dot{\kappa}\Big[\Delta_{T0} + \mu v_e - \frac{1}{2}P_2(\mu)(\Delta_{T2} + \Delta_{P0} + \Delta_{P2})\Big]\Big], \tag{6}$$

where the optical depth is $\kappa \equiv -\int_\tau^{\tau_0} d\tau' \dot{\kappa}(\tau') < 0$ and the differential optical depth is $\dot{\kappa} = an_e\sigma_T$, which is the common factor for all collision terms. Here n_e is the number density of *free* electrons in coordinate space and $\sigma_T = 6.65 \times 10^{-25}$ cm^2 is the Thomson cross section. $\mu \equiv (\hat{k}\cdot\hat{n})$, $\alpha \equiv \frac{\dot{h}+6\dot{\eta}}{2k^2}$ and v_e is the electron velocity. Suffix P means polarization mode and integer suffix labels multipoles of spherical harmonics.

In DDM model, metric perturbations h and η get significant contribution from the daughter particles. The perturbation evolution of the daughter particle has the same series with the massive neutrino

$$\dot{\Psi}_0 = -\frac{qk}{\epsilon}\Psi_1 + \frac{1}{6}\dot{h}\frac{d\ln f}{d\ln q}, \tag{7a}$$

$$\dot{\Psi}_1 = \frac{qk}{\epsilon}\left(\Psi_0 - \frac{2}{3}\Psi_2\right), \tag{7b}$$

$$\dot{\Psi}_2 = \frac{qk}{\epsilon}\left(\frac{2}{5}\Psi_1 - \frac{3}{5}\Psi_3\right) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f}{d\ln q}, \tag{7c}$$

$$\dot{\Psi}_\ell = \frac{qk}{\epsilon}\frac{1}{2\ell+1}\left(\ell\Psi_{\ell-1} - (\ell+1)\Psi_{\ell+1}\right), \quad \ell \geqslant 3, \tag{7d}$$

$$\dot{\Psi}_\ell = \frac{qk}{\epsilon}\Psi_{\ell-1} + \frac{\ell+1}{\tau}\Psi_\ell, \quad \text{as truncation.} \tag{7e}$$

The right-hand side source term of Eq. (4) have contributions only from the first three perturbation modes ($\delta\rho, \delta p \propto \Psi_0$, $\theta \propto \Psi_1$ and $\sigma \propto \Psi_2$). Because all particle species such as baryon, cold dark matter (CDM), photon, massless as well as massive neutrino talk to gravity, all their evolution will be modified.

The codes also calculate the transfer functions (TF) as an intermediate step in the CMB calculations, which is an indication of the large scale structure. The TF is by definition the normalized ratio of perturbation growth factor, from an very early stage to certain late stage, the normalization is taken with a very large scale which is out of horizon in the whole evolution

$$T(k) \equiv \frac{\delta(k,t_f)/\delta(k,t_i)}{\delta(k\to 0,t_f)/\delta(k\to 0,t_i)}. \tag{8}$$

3. Modification

We introduce free parameter Ω_d which corresponds to the daughter particles' energy density *today*, while still keeping the nondecay CDM part Ω_c. In this way we can treat any combination of decaying and nondecay dark matter. For convenience to use Eq. (1), Ω_d is not the whole energy density but only the mass contribution to energy density, namely the kinetic energy of the daughter particle is not included. Therefore we should have at least one massive daughter particle for this parameterization. Except for an extreme relativistic decay, the kinetic energy contribution is small and Ω_d represents the energy density very well. We only consider the one-to-two decay process and introduce two mass ratios $\frac{m_{p1}}{m_o}$ and $\frac{m_{p2}}{m_o}$, where m_{p1} and m_{p2} are separately the masses of two product particles and m_o is the mass of original particle. The last free parameter is the decay lifetime T_d. So the complete set of new parameters includes Ω_d, T_d, $\frac{m_{p1}}{m_o}$ and $\frac{m_{p2}}{m_o}$.

Let us go through what will be modified in our DDM model, compared with the standard ΛCDM universe. First, the decay process will affect the expansion of the universe, through changing the equation of state. Before decay the DDM behaves as the CDM with a constant equation of state $\omega = 0$, while after decay it does not hold and the momentum and energy of daughter particle is subject to redshift, as what happens to massive neutrino. Since before decay the DDM particle can be approximated as being at rest, given the masses of the parent and daughter particles the initial transverse momentum p_T is fixed for a two body decay. The comoving momentum for each individual daughter particle is determined only by the scale factor a^* at which the decay happens (in this Letter we will always use a $*$ to denote the quantity right at decay). As the scale factor $a(\tau)$ grows the physical momentum $p(\tau)$ decreases, while preserving

$$q = a(\tau)p(\tau) = a^* p_T. \tag{9}$$

With this relation the energy density and pressure can be evaluated numerically for product particles in the modification, so is the expansion process.

The decay is a continuous process for the set of DDM particles, the way for our numerical study is to discretize it into $30 \sim 60$ channels, by which we can achieve 0.5% precision for CMB peak height. Each channel corresponds to daughter particles produced in a small scale factor region $a^* \sim a^* + da^*$ and has its own Ψ_ℓ series. As the universe expands the channels are gradually filled channel by channel in order. The perturbation evolution is described by Eq. (7), the only subtlety comes through the factor $\frac{d\ln f}{d\ln q}$: the unperturbed distribution $f(q)$ is no longer the Fermi–Dirac distribution of neutrino, but determined by the decay process. A number of product particles proportional to $d\Omega$ will be redistributed into q space $d^3q = 4\pi q^2\,dq = 4\pi p_T^3 a^{*2}\,da^*$, so up to some factor the unperturbed partition function from decay is

$$f \equiv \frac{dn}{d^3p} \propto \frac{d\Omega}{d^3q} = \frac{e^{-\frac{t}{T_d}}\frac{dt}{T_d}}{4\pi p_T^3 a^{*2}\,da^*} = \frac{e^{-\frac{t}{T_d}}}{4\pi T_d p_T^3 a^* \dot{a}^*}, \tag{10}$$

and $\frac{d\ln f}{d\ln q}$ can be calculated

$$\frac{d\ln f}{d\ln q} = -\frac{a^{*2}}{T_d\dot{a}^*} - \frac{3}{2} + \frac{3\bar{p}^*}{2\bar{\rho}^*}. \tag{11}$$

The last issue for the perturbation evolution differential equation set is the initial condition. The initial values of all perturbation modes should naturally inherit the values before decay, which for DDM they are the same as CDM and all higher multipoles vanish. So we have

$$\Psi_0^* = \delta_{\text{CDM}}, \tag{12}$$

$$\Psi_\ell^* = 0, \quad \ell \geqslant 1. \tag{13}$$

Including the daughter particles' contribution to metric perturbation finishes our modification.