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Calculating contracted tensor Feynman integrals

J. Fleischer [a], T. Riemann [b,*]

[a] Fakultät für Physik, Universität Bielefeld, Universitätsstr. 25, 33615 Bielefeld, Germany
[b] Deutsches Elektronen-Synchrotron, DESY, Platanenallee 6, 15738 Zeuthen, Germany

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ABSTRACT

A recently derived approach to the tensor reduction of 5-point one-loop Feynman integrals expresses the tensor coefficients by scalar 1-point to 4-point Feynman integrals completely algebraically. In this Letter we derive extremely compact algebraic expressions for the contractions of the tensor integrals with external momenta. This is based on sums over signed minors weighted with scalar products of the external momenta. With these contractions one can construct the invariant amplitudes of the matrix elements under consideration, and the evaluation of one-loop contributions to massless and massive multi-particle production at high energy colliders like LHC and ILC is expected to be performed very efficiently.

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1. Introduction

In a recent article [1] (hereafter quoted as Ref. I), we have worked out an algebraic method to present one-loop tensor integrals in terms of scalar one-loop 1-point to 4-point functions. The tensor integrals are defined as

$$I_n^{\mu_1\cdots\mu_R} = \int \frac{d^dk}{i\pi^{d/2}} \frac{\prod_{r=1}^R k^{\mu_r}}{\prod_{j=1}^n c_j}, \tag{1.1}$$

with denominators c_j, having *chords* q_j,

$$c_j = (k-q_j)^2 - m_j^2 + i\varepsilon. \tag{1.2}$$

Here, we use the generic dimension $d = 4-2\varepsilon$. The central problem are the 5-point tensor functions. We derived algebraic expressions for them in terms of *higher-dimensional* scalar 4-point functions with *raised indices* (powers of the scalar propagators). There are several ways to go. One option is to avoid the appearance of inverse Gram determinants $1/()_5$. For rank $R=5$, e.g.,

$$I_5^{\mu\nu\lambda\rho\sigma} = \sum_{s=1}^{5}\left[\sum_{i,j,k,l,m=1}^{5} q_i^\mu q_j^\nu q_k^\lambda q_l^\rho q_m^\sigma E_{ijklm}^s + \sum_{i,j,k=1}^{5} g^{[\mu\nu} q_i^\lambda q_j^\rho q_k^{\sigma]} E_{00ijk}^s + \sum_{i=1}^{5} g^{[\mu\nu} g^{\lambda\rho} q_i^{\sigma]} E_{0000i}^s\right], \tag{1.3}$$

see equations (I.4.60), (I.4.61). The tensor coefficients are expressed in terms of integrals $I_{4,i\cdots}^{[d+]^l,s}$, e.g. according to (I.4.62):

$$E_{ijklm}^s = -\frac{1}{\binom{0}{0}_5}\left\{\left[\binom{0l}{sm}_5 n_{ijk} I_{4,ijk}^{[d+]^4,s} + (i \leftrightarrow l) + (j \leftrightarrow l) + (k \leftrightarrow l)\right] + \binom{0s}{0m}_5 n_{ijkl} I_{4,ijkl}^{[d+]^4,s}\right\}. \tag{1.4}$$

The scalar integrals are

$$I_{p,ijk\cdots}^{[d+]^l,stu\cdots} = \int \frac{d^{[d+]^l}k}{i\pi^{[d+]^l/2}} \prod_{r=1}^{n} \frac{1}{c_r^{1+\delta_{ri}+\delta_{rj}+\delta_{rk}+\cdots-\delta_{rs}-\delta_{rt}-\delta_{ru}-\cdots}}, \tag{1.5}$$

* Corresponding author.
E-mail addresses: fleischer@physik.uni-bielefeld.de (J. Fleischer), Tord.Riemann@desy.de (T. Riemann).

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where p is the number of internal lines and $[d+]^l = 4 - 2\varepsilon + 2l$. Further, we use the notations of signed minors (I.2.14). At this stage, the higher-dimensional 4-point integrals still depend on tensor indices, namely through the indices i, j etc. The most complicated explicit example $I_{4,ijkl}^{[d+]^4,s}$ appears in (1.4). Now, in a next step, one may avoid the appearance of inverse sub-Gram determinants $()_4$. Indeed, after tedious manipulations, one arrives at representations in terms of scalar integrals $I_4^{[d+]^l}$ plus simpler 3-point and 2-point functions, and the complete dependence on the indices i of the tensor coefficients is contained now in the pre-factors with signed minors. One can say that the indices *decouple* from the integrals. As an example, we reproduce the 4-point part of (I.5.21),

$$\begin{aligned}
n_{ijkl} I_{4,ijkl}^{[d+]^4} &= \frac{\binom{0}{i}}{\binom{0}{0}} \frac{\binom{0}{j}}{\binom{0}{0}} \frac{\binom{0}{k}}{\binom{0}{0}} \frac{\binom{0}{l}}{\binom{0}{0}} d(d+1)(d+2)(d+3) I_4^{[d+]^4} \\
&+ \frac{\binom{0i}{0j}\binom{0}{k}\binom{0}{l} + \binom{0i}{0k}\binom{0}{j}\binom{0}{l} + \binom{0j}{0k}\binom{0}{i}\binom{0}{l} + \binom{0i}{0l}\binom{0}{j}\binom{0}{k} + \binom{0j}{0l}\binom{0}{i}\binom{0}{k} + \binom{0k}{0l}\binom{0}{i}\binom{0}{j}}{\binom{0}{0}^3} d(d+1) I_4^{[d+]^3} \\
&+ \frac{\binom{0i}{0l}\binom{0j}{0k} + \binom{0j}{0l}\binom{0i}{0k} + \binom{0k}{0l}\binom{0i}{0j}}{\binom{0}{0}^2} I_4^{[d+]^2} + \cdots
\end{aligned} \tag{1.6}$$

In (1.6), one has to understand the 4-point integrals to carry the corresponding index s and the signed minors are $\binom{0}{k} \to \binom{0s}{ks}_5$ etc. This type of relations may be called "recursion relations for small Gram determinants".

In an alternative treatment, tensor reduction formulas free of $g^{\mu\nu}$ terms were derived in Ref. I. In that case, inverse powers of $()_5$ are tolerated. The most involved object studied was (I.3.20):

$$I_5^{\mu\nu\lambda\rho\sigma} = I_5^{\mu\nu\lambda\rho} \cdot Q_0^{\sigma} - \sum_{s=1}^{5} I_4^{\mu\nu\lambda\rho,s} \cdot Q_s^{\sigma}, \tag{1.7}$$

with the 4-point tensor functions (I.3.29) and (I.3.18)

$$I_4^{\mu\nu\lambda\rho,s} = Q_0^{s,\mu} Q_0^{s,\nu} Q_0^{s,\lambda} Q_0^{s,\rho} I_4^s + 3 \frac{()_5^2}{\binom{s}{s}_5^2} Q_s^{\mu} Q_s^{\nu} Q_s^{\lambda} Q_s^{\rho} \cdot I_4^{[d+]^2,s} + \frac{()_5}{\binom{s}{s}_5} Q_s^{[\mu} Q_s^{\nu} J_4^{\lambda,s} Q_0^{s,\rho]} + \cdots, \tag{1.8}$$

$$J_4^{\mu,s} = -Q_0^{s,\mu} I_4^{[d+]} + \sum_{t=1}^{5} Q_t^{s,\mu} I_3^{[d+],st}. \tag{1.9}$$

The dots in (1.8) indicate 3-point functions, and

$$Q_s^{\mu} = \sum_{i=1}^{5} q_i^{\mu} \frac{\binom{s}{i}_5}{()_5}, \quad s = 0, \dots, 5, \tag{1.10}$$

$$Q_s^{t,\mu} = \sum_{i=1}^{5} q_i^{\mu} \frac{\binom{st}{it}_5}{\binom{t}{t}_5}, \quad s, t = 1, \dots, 5. \tag{1.11}$$

Also here, the tensor coefficients have been represented by scalar functions free of tensor indices.

We remark that all the above-mentioned results are due to a systematic application of methods described and developed in [2]. For the present Letter the "algebra of signed minors" [3] plays a particularly important role. This method was also used in [2] and further developed to its full power in [4]. In the latter article also 6-point functions have been treated on this basis.

In the next section we will develop a very efficient method to evaluate realistic matrix elements with tensor integral representations of the above kind.

2. Contracting the tensor integrals

To apply the approach most efficiently one should construct projection operators for the invariant amplitudes of the matrix elements under consideration. These projectors, of course, depend on the tensor basis and have to be constructed for each process specifically. If done like that, the tensor indices of the loop integrals are saturated by contractions with external momenta p_r. The chords in (1.2) are given in terms of the external momenta as $q_i = -(p_1 + p_2 + \cdots + p_i)$, with $q_n = 0$, and inversely $p_r = q_{r-1} - q_r$. Then, any of the integrals to be evaluated is a simple linear combination of integrals containing products with chords, $(q_r \cdot k)$:

$$q_{i_1\mu_1} \cdots q_{i_R\mu_R} I_5^{\mu_1 \cdots \mu_R} = \int \frac{d^d k}{i\pi^{d/2}} \frac{\prod_{r=1}^{R} (q_{i_r} \cdot k)}{\prod_{j=1}^{5} c_j}. \tag{2.1}$$

There is another type of external vectors, i.e. the polarization vectors ε_i of spin-1 bosons. They, however, are taken into account in the definition of the tensor structure of the matrix elements in terms of scalar products $(\varepsilon_i \cdot p_j)$ with some external momenta p_j. The same applies to contractions with γ matrices $\not\varepsilon_i$ and $\not p_i$ in spinor chains. Thus, polarization vectors and γ matrices will not show up in the sums one has to perform.

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