



Second-order phase transition of Kehagias–Sfetsos black hole in deformed Hořava–Lifshitz gravity

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ABSTRACT

We study the second-order phase transition (SOPT) for the spherically symmetric Kehagias–Sfetsos (KS) black hole in the deformed Hořava–Lifshitz gravity by applying the methods of equilibrium and non-equilibrium fluctuations. We find that, although the KS black hole has only one mass parameter as the usual Schwarzschild ones, the SOPT will take place if the mass of the KS black hole changes across the critical point $\frac{\sqrt{5+\sqrt{33}(\sqrt{33}-1)}}{16\sqrt{\omega}}$. The result show us that there is difference between the Hořava–Lifshitz gravity and the Einstein's gravity theory.

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1. Introduction

It is well known that the non-renormalizable of gravity is a large challenge for Einstein's gravity theory. Recently, Hořava [1–3] proposed a new class of quantum gravity which is non-relativistic and power-counting renormalizable. The key feature of this theory is that the space and time exhibit Lifshitz scale invariance $t \rightarrow b^z t$ and $x_i \rightarrow b x_i$ with $z > 1$. It is this anisotropic rescaling that makes Hořava's theory power-counting renormalizable. In the IR region such a Hořava's theory can reduce to the well-known Einstein gravity. Therefore, a lot of attention has been focused on this gravity theory [4–19].

In general, the IR vacuum in Hořava's theory is anti-de Sitter (AdS) spacetimes. In order to obtain a Minkowski vacuum in the IR sector, one can add the term " $\mu^4 R$ " in the action and then take the $\Lambda_W \rightarrow 0$ limit. This does not change the UV properties of the theory, but it alters the IR properties. Making use of such a deformed action, Kehagias et al. [10] obtain the asymptotic flat spherically symmetric vacuum black hole solution which has two event horizons. The heat capacity is positive for the small KS black hole and it is negative for the large one. It means that the small KS black holes are the stable in the Hořava's theory. The result imply that there exists the distinct differences between the KS black hole and the Schwarzschild black hole.

The investigation of SOPT of black hole is helpful to explore the black hole's property [20–22]. To the best of our knowledge, there is no report about investigation for SOPT of the KS black hole in the deformed Hořava–Lifshitz gravity. On the other hand, there is a paradox about where the SOPT is taken place for a long time. Some authors [23–26] argue that the SOPT is taken place when $C \rightarrow \infty$ by applying thermodynamical equilibrium fluctuations. The other [27] calculated the non-equilibrium fluctuation of mass and entropy and found that these fluctuations diverge when $r_+ \rightarrow r_-$ and they are finite when $C \rightarrow \infty$. So they put forward that the SOPT of KS black hole takes place where $r_+ \rightarrow r_-$ rather than where $C \rightarrow \infty$. In this Letter, we will address these question carefully by studying the SOPT of the KS black hole [10] in the deformed Hořava–Lifshitz gravity using the methods of equilibrium and non-equilibrium fluctuations.

The Letter is organized as follows. In Section 2, we give a brief description of solution in the deformed Hořava–Lifshitz black hole spacetime. In Section 3, we calculate a SOPT point by using the method of equilibrium fluctuations. In Section 4, we make use of the non-equilibrium thermodynamic fluctuations and find a SOPT point that is just the same point as we find in Section 3. We present our conclusions and make some discussions in the last section. Throughout, we shall set $c = G = \hbar = k = 1$.

2. Rigorous solution in deformed Hořava–Lifshitz gravity

In the Hořava theory, a deformed action of the non-relativistic renormalizable gravitational theory is given by [10]

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$$S_{HL} = \int dt d^3x (\mathcal{L}_0 + \tilde{\mathcal{L}}_1),$$

$$\mathcal{L}_0 = \sqrt{g}N \left\{ \frac{2}{\kappa^2} (K_{ij}K^{ij} - \lambda K^2) + \frac{\kappa^2 \mu^2 (\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)} \right\}, \quad (1)$$

$$\begin{aligned} \tilde{\mathcal{L}}_1 = \sqrt{g}N \left\{ \frac{\kappa^2 \mu^2 (1-4\lambda)}{32(1-3\lambda)} R^2 - \frac{\kappa^2}{2w^4} \right. \\ \left. \times \left(C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left(C^{ij} - \frac{\mu w^2}{2} R^{ij} \right) + \mu^4 R \right\}, \quad (2) \end{aligned}$$

where κ^2 , λ , μ , w and Λ_W are constant parameters, K_{ij} is extrinsic curvature in the (3+1)-dimensional ADM formalism

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i), \quad (3)$$

and C_{ij} is the Cotton tensor

$$C^{ij} = \epsilon^{ik\ell} \nabla_k \left(R_\ell^{(3)j} - \frac{1}{4} R^{(3)} \delta_\ell^j \right). \quad (4)$$

Taking the $\Lambda_W \rightarrow 0$ limit and letting $\lambda = 1$, it was found that the speed of light, the Newton constant are described by the following relations [10]

$$c^2 = \frac{\kappa^2 \mu^4}{2}, \quad G = \frac{\kappa^2}{32\pi c}. \quad (5)$$

A static and asymptotically flat KS black hole was found in [10] which has the following form

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (6)$$

with

$$f(r) = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)} \quad (7)$$

where $\omega = \frac{16\mu^2}{\kappa^2}$ and M is an integration constant related to the mass of the KS black hole.

The outer (inner) horizons are given by

$$r_{\pm} = M \pm \sqrt{M^2 - \frac{1}{2\omega}}. \quad (8)$$

The corresponding Hawking temperature and heat capacity are [28]

$$T_H = \frac{1}{4\pi} \partial_r f(r)|_{r=r_+} = \frac{\omega(r_+ - M)}{2\pi(1 + \omega r_+^2)} = \frac{2\omega r_+^2 - 1}{8\pi(\omega r_+^3 + r_+)}, \quad (9)$$

$$C = \frac{\partial M}{\partial T} = -\frac{2\pi}{\omega} \frac{(1 + \omega r_+^2)^2 (2\omega r_+^2 - 1)}{2\omega^2 r_+^4 - 5\omega r_+^2 - 1}. \quad (10)$$

3. Equilibrium fluctuation for KS black hole in deformed gravity

In thermodynamical fluctuational theory, we always express fluctuations of thermodynamical quantities in an equilibrium system as some mean square fluctuations. If the mean square fluctuations are divergent at a point, we call the point as a phase transition point for the thermodynamical system. If the entropy of the system is continuous at the point, we name the phase transition as the SOPT. Therefore, we can look for the SOPT point in a thermodynamical system according to singularity of fluctuations of the thermodynamical quantities and continuity of the entropy.

We suppose that the KS black hole [10] is initially at temperature T_o (the point o is arbitrary). The variation of temperature due to thermal fluctuation of the mass is $\Delta\epsilon T_o$, where $|\Delta\epsilon| \leq B$

and B being a sufficiently small constant. From the first law of the thermodynamics we have

$$\Delta M = T_o(1 + \Delta\epsilon)\Delta S \approx T_o\Delta S \quad (11)$$

where T_o is determined by (9).

The fluctuation probability p is

$$p \propto \exp \Delta S \sim \exp \left(\frac{\Delta M}{T_o} \right). \quad (12)$$

We know from the definition of heat capacity that

$$\Delta M = C T_o \Delta\epsilon \quad (13)$$

where heat capacity C is determined by (10).

Because $|\Delta\epsilon| \leq B$, the variation of the mass ΔM may be expressed as [25]

$$|\Delta M| \leq |C| T_o B. \quad (14)$$

With Eqs. (12), (14) and the condition of normalized

$$\int_{-B|C|T_o}^{B|C|T_o} p d(\Delta M) = 1, \quad (15)$$

we find that the fluctuation probability p can be rewritten as

$$p = \frac{1}{2T_o \sinh(B|C|)} \exp \left(\frac{\Delta M}{T_o} \right). \quad (16)$$

Making use of Eqs. (14) and (16), we can get the mean square fluctuation of M

$$\begin{aligned} \langle (\Delta M)^2 \rangle &= \frac{1}{2T_o \sinh(B|C|)} \int_{-B|C|T_o}^{B|C|T_o} (\Delta M)^2 \exp \left(\frac{\Delta M}{T_o} \right) d(\Delta M) \\ &= T_o^2 (2 + B^2 C^2 - 2B|C| \coth(B|C|)). \end{aligned} \quad (17)$$

In the same way, we obtain

$$\langle (\Delta S)^2 \rangle = \frac{1}{T_o^2} \langle (\Delta M)^2 \rangle = 2 + B^2 C^2 - 2B|C| \coth(B|C|), \quad (18)$$

$$\langle (\Delta T)^2 \rangle = \frac{1}{|C|^2} \langle (\Delta M)^2 \rangle = \frac{T_o^2}{C^2} (2 + B^2 C^2 - 2B|C| \coth(B|C|)). \quad (19)$$

We observe from Eqs. (9), (10), (17) and (18) that $\langle (\Delta S)^2 \rangle \rightarrow \infty$ and $\langle (\Delta M)^2 \rangle \rightarrow \infty$ when $C \rightarrow \infty$. In other words, at the point $C \rightarrow \infty$, even if we add the small perturbation of M to black hole, the fluctuations of mass and entropy of the KS black hole will become so large that we can't describe it with a normal equilibrium state. Thus, when the heat capacity changes from positive to negative, we think that the KS black hole changes from a phase into another phase and the critical point lies at $\frac{\sqrt{5+\sqrt{33}}(\sqrt{33}-1)}{16\sqrt{\omega}}$. It is a SOPT point because the entropy of the KS black hole is continuous at this critical point. On the other hand, when $r_+ \rightarrow r_-$, Eqs. (17) and (18) lead to $\langle (\Delta S)^2 \rangle \rightarrow 0$, $\langle (\Delta M)^2 \rangle \rightarrow 0$. It means that when the KS black hole has a small deviation from equilibrium state, all thermodynamic fluctuations goes to zero under the condition that $r_+ \rightarrow r_-$, and then the KS black hole will come back to the initial equilibrium state. So we think the SOPT of the KS black hole in the deformed Hořava–Lifshitz gravity doesn't occur as $r_+ \rightarrow r_-$ but $C \rightarrow \infty$.

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