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Observational information for $f(T)$ theories and dark torsion

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ABSTRACT

In the present work we analyze and compare the information coming from different observational data sets in the context of a sort of $f(T)$ theories. We perform a joint analysis with measurements of the most recent type Ia supernovae (SNe Ia), Baryon Acoustic Oscillation (BAO), Cosmic Microwave Background radiation (CMB), Gamma-Ray Bursts data (GRBs) and Hubble parameter observations (OHD) to constraint the only new parameter these theories have. It is shown that when the new combined BAO/CMB parameter is used to put constraints, the result is different from previous works. We also show that when we include Observational Hubble Data (OHD) the simpler ΛCDM model is excluded to one sigma level, leading the effective equation of state of these theories to be of phantom type. Also, analyzing a tension criterion for SNe Ia and other observational sets, we obtain more consistent and better suited data sets to work with these theories.

1. Introduction

Current cosmological observations, mainly from type Ia supernovae, show that the universe is undergoing accelerated expansion [1–4]. This accelerated expansion has been attributed to a dark energy component with negative pressure. The simplest explanation for this dark energy seems to be the cosmological constant. However, among many candidates [5–7], some modified gravity models have also been proposed based on, for example, $f(R)$ theories [8–19].

Some models based on modified teleparallel gravity were presented as an alternative to inflationary models [20,21] or showing a cosmological solution for the acceleration of the universe by means of a sort of theories of modified gravity, namely $f(L_T)$ [22], based on a modification of the Teleparallel Equivalent of General Relativity (TEGR) Lagrangian [23,24] where *dark torsion* is the responsible for the observed acceleration of the universe, and the field equations are always 2nd order equations. It was shown in [22] that this fact makes these theories simpler than the dynamical equations resulting in $f(R)$ theories among other advantages. Recently, in [25] this sort of modified gravity theories was called $f(T)$ theories and some works have begun to develop in this area [26–35].

In [36] the tension and systematics in the Gold06 SNe Ia data set have been investigated in great detail. Other authors, working with different SNe Ia sets found these were in tension with other SNe Ia sets and also with BAO and CMB [37,38]. In [37], analyzing the Union data set [2], the UnionT truncated data set was built by discarding the supernovae generating the tension by using the ΛCDM model to select the outliers. In [38], performing the same truncation procedure of [37] for 10 different models, it was suggested that the impact of different models would be negligible.

In this work we present thorough observational information useful to work with $f(T)$ theories by using the latest Union2 SNe Ia compilation released [3], the new combined parameter from Baryon Acoustic Oscillation and Cosmic Microwave Background radiation (BAO/CMB) [39] (more suitable for non-standard models than the usually used R and A parameters), a Gamma-Ray Burst data set [40] and constraints from Observational Hubble Data (OHD) [41–43].

This Letter is organized as follows: in Section 2 we review the fundamental concepts about $f(T)$ theories to, in Section 3, analyze a criterion of tension to improve the study of the new data sets including BAO/CMB and GRBs. In Section 4 we perform the truncation of Union2 calculating the relative deviation to the best fit of the $f(T)$ prediction for each one of the 557 points following [37,38] in order to show the disappearing of tension and establishing a new set suitable for $f(T)$ theories. In Section 5 we add the OHD observational information and discuss some remarkable results and, in Section 6, we summarize the conclusions of this work.

2. General considerations about $f(T)$ theories

Teleparallelism [23,24] uses as dynamical object a vierbein field $\mathbf{e}_i(x^\mu)$, $i = 0, 1, 2, 3$, which is an orthonormal basis for the tangent space at each point x^μ of the manifold: $\mathbf{e}_i \cdot \mathbf{e}_j = \eta_{ij}$, where

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$\eta_{ij} = \text{diag}(1, -1, -1, -1)$. Each vector $\mathbf{e}_i$ can be described by its components e_i^μ, $\mu = 0, 1, 2, 3$, in a coordinate basis; i.e. $\mathbf{e}_i = e_i^\mu \partial_\mu$. Notice that Latin indices refer to the tangent space, while Greek indices label coordinates on the manifold. The metric tensor is obtained from the dual vierbein as $g_{\mu\nu}(x) = \eta_{ij} e_\mu^i(x) e_\nu^j(x)$. Differing from General Relativity (GR), which uses the torsionless Levi-Civitá connection, Teleparallelism uses the curvatureless Weitzenböck connection [44], whose non-null torsion is

$$T^\lambda_{\mu\nu} = \hat{\Gamma}^\lambda_{\nu\mu} - \hat{\Gamma}^\lambda_{\mu\nu} = e_i^\lambda \left(\partial_\mu e_\nu^i - \partial_\nu e_\mu^i\right) \tag{1}$$

The TEGR Lagrangian is built with the torsion (1), and its dynamical equations for the vierbein imply the Einstein equations for the metric. The teleparallel Lagrangian is [24,45,46],

$$L_T \equiv T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu} \tag{2}$$

where:

$$S_\rho{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\theta\nu}{}_\theta - \delta^\nu_\rho T^{\theta\mu}{}_\theta\right) \tag{3}$$

and $K^{\mu\nu}{}_\rho$ is the contorsion tensor:

$$K^{\mu\nu}{}_\rho = -\frac{1}{2}\left(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}\right) \tag{4}$$

which equals the difference between Weitzenböck and Levi-Cività connections.

For a flat homogeneous and isotropic Friedmann–Robertson–Walker universe (FRW),

$$e^i_\mu = \text{diag}\big(1, a(t), a(t), a(t)\big) \tag{5}$$

where $a(t)$ is the cosmological scale factor. By replacing in (1), (3) and (4) one obtains

$$T = S^{\rho\mu\nu} T_{\rho\mu\nu} = -6\frac{\dot{a}^2}{a^2} = -6H^2 \tag{6}$$

H being the Hubble parameter $H = \dot{a}a^{-1}$.

In these modified gravity theories, the action is built promoting T to a function $f(T)$. The case $f(T) = T$ corresponds to TEGR. In an $f(T)$ theory the spinless matter couples to the metric in the standard form. Therefore, the equations of a freely falling particle are the equations of the geodesics. Moreover, the source in the equations for the geometry results to be the matter energy–momentum tensor. In these aspects there is no difference with GR. If matter is distributed isotropically and homogeneously, the metric is the FRW metric and all kinematic equations (luminosity distance, angular distance, cosmological redshift, etc.) will be identical to the GR case. Any modification in the null geodesics followed by light rays will be exclusively in the scale factor $a(t)$. Some authors have mentioned that $f(T)$ theories are not invariant under local Lorentz transformations [20,34]. However, if this would affect the viability of these models is a subject which is currently being analyzed.

The variation of the action with respect to the vierbein leads to the field equations,

$$e^{-1}\partial_\mu\left(e S_i{}^{\mu\nu}\right) f'(T) - e_i{}^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} f'(T) + S_i{}^{\mu\nu}\partial_\mu(T) f''(T) + \frac{1}{4} e_i^\nu f(T) = 4\pi G e_i{}^\rho T_\rho{}^\nu \tag{7}$$

where a prime denotes differentiation with respect to T, $S_i{}^{\mu\nu} = e_i{}^\rho S_\rho{}^{\mu\nu}$ and $T_{\mu\nu}$ is the matter energy–momentum tensor.

The substitution of the vierbein (5) in (7) for $i = 0 = \nu$ yields

$$12H^2 f'(T) + f(T) = 16\pi G\rho \tag{8}$$

Besides, the equation $i = 1 = \nu$ is

$$48H^2 f''(T)\dot{H} - f'(T)\big[12H^2 + 4\dot{H}\big] - f(T) = 16\pi G p \tag{9}$$

In Eqs. (8)–(9), $\rho(t)$ and $p(t)$ are the total density and pressure respectively.

In [22] it was shown that when $f(T)$ is a power law such as

$$f(T) = T - \frac{\alpha}{(-T)^n} \tag{10}$$

leads to reproduce the observed accelerated expansion of the universe, being α and n real constants to be determined by observational constraints.

From (8) along with (10), the modified Friedmann equation results to be (e.g. [22])

$$H^2 - \frac{(2n+1)\alpha}{6^{n+1} H^{2n}} = \frac{8}{3}\pi G\rho \tag{11}$$

where $\rho = \rho_{mo}(1+z)^3 + \rho_{ro}(1+z)^4$, z is the cosmological redshift and as it is usual, we will call $\Omega_i = 8\pi G\rho_{io}/(3H_0^2)$ to the contributions of matter and radiation to the total energy density today. For $\alpha = 0$ the GR spatially flat Friedmann equation is retrieved. The case $n = 0$ recovers the GR dynamics with cosmological constant. Compared with GR, n is the sole new free parameter (see [22] for details).

In the next sections, we will use a $\chi^2 = \chi^2_{SNe} + \chi^2_{BAO/CMB} + \chi^2_{GRB} + \chi^2_{OHD}$ statistic to find best fits for the free parameters Ω_m and n of a model given by (10) using several data sets. The separate χ^2 of SNe Ia, BAO/CMB, GRBs and OHD and the corresponding data sets used in this work are shown in Appendix B. In order to see whether our model is favored over the ΛCDM model, we will also use the information criterion known as *AIC* (Akaike Information Criterion) [47,48]. The *AIC* is defined as $AIC = -2\ln\mathcal{L}_{\max} + 2k$, where the likelihood is defined as $\mathcal{L} \propto e^{\chi^2/2}$, the term $-2\ln\mathcal{L}_{\max}$ corresponds to the $\chi^2_{\min}$ and k is the number of parameters of the model. According to this criterion a model with the smaller *AIC* is considered to be the best, and a difference $|\Delta AIC|$ in the range between 0 and 2 means that the two models have about the same support from the data. For a difference between 2 and 4 this support is considerably less for the model with the larger *AIC*, while for a difference >10 the model with the larger *AIC* is practically irrelevant [49].

3. Constraining dark torsion with updated data sets

We found interesting to analyze what would happen if we applied a criterion in order to study the consistency between data sets, a criterion more restrictive than the only fact that the confidence intervals overlap. To perform this analysis, we adopted the criterion of considering the existence of tension between a given data set and another set constituted combining several data sets (including the first one) as the fact that the best fit point to the first data set is out of the 68.3% (1σ) confidence level contour given by the combined data set. Similar criteria were adopted in their analysis by [36–38]. One could choose not to use this more restrictive criterion; however, we wanted to investigate its consequences of applying it to several data sets in the framework of $f(T)$ theories. In [36], for example, the best fits to sets and subsets of SNe are compared with the means of determining if two of those are in tension or not, and how far from the confidence intervals lies the ΛCDM model. With our adopted criterion, we seek more physical consistency between best-fits, so the best fits do not drive to too different cosmological evolutions. The best fit which effective equation of state is of the phantom type [50] ($w_{eff} < -1$) tells us about very different physics from the one that is not. Also,