Physics Letters B 695 (2011) 501–506

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Higher equations of motion in $N = 2$ superconformal Liouville field theory

Changrim Ahn [a,b,*], Marian Stanishkov [b,1], Michail Stoilov [c]

[a] Department of Physics, Ewha Womans University, Seoul 120-750, South Korea
[b] Institute for the Early Universe, Ewha Womans University, Seoul 120-750, South Korea
[c] Institute for Nuclear Research and Nuclear Energy (INRNE), Bulgarian Academy of Sciences (BAS), 1784 Sofia, Bulgaria

ARTICLE INFO

Article history:
Received 28 October 2010
Accepted 26 November 2010
Available online 1 December 2010
Editor: L. Alvarez-Gaumé

In memory of Alyosha Zamolodchikov

Keywords:
$N = 2$ supersymmetry
Liouville field theory
Equations of motion

ABSTRACT

We present an infinite set of higher equations of motion in $N = 2$ supersymmetric Liouville field theory. They are in one to one correspondence with the degenerate representations and are enumerated in addition to the $U(1)$ charge ω by the positive integers m or (m, n) respectively. We check that in the classical limit these equations hold as relations among the classical fields.

In Ref. [1] it has been shown that in the Liouville field theory (LFT) an infinite set of relations holds for quantum operators. These equations relate different basic Liouville primary fields $V_\alpha(z)$ (V_α can be thought of as normal ordered exponential field $\exp(\alpha\phi)$ of the basic Liouville field ϕ). They are parameterized by a pair of positive integers (m, n) and are called conventionally "higher equations of motion" (HEM), because the first one $(1, 1)$ coincides with the usual Liouville equation of motion. The equations are derived on the basis of a conjecture of the vanishing of all singular vectors, imposed by the requirement of irreducibility of the corresponding representation. They are easily verified in the classical LFT. Higher equations turn out to be useful in practical calculations. In particular, in [2–5], they were used to derive general four-point correlation function in the minimal Liouville gravity.

Similar operator valued relations have been found also for $N = 1$ supersymmetric Liouville field theory (SLFT) [6] and for $SL(2, R)$ Wess–Zumino–Novikov–Witten model [7,8]. Recently it was shown in [9] that such relations hold for the boundary operators in the LFT with conformal boundary.

It is the purpose of this Letter to reveal a similar set of higher equations of motion in $N = 2$ SLFT. The $N = 2$ SLFT has a wide variety of applications in string theory [10–12]. This theory is quite interesting because of the fact that it has actually few properties in common with the $N = 0, 1$ SLFTs. For example, unlike the Liouville theories with less supersymmetry, the $N = 2$ SLFT does not have a simple strong–weak coupling duality. In fact, under the change $b \to 1/b$ of the coupling constant, the $N = 2$ SLFT flows to another $N = 2$ supersymmetric theory as proposed in [13,14]. Another important difference between the $N = 2$ SLFT and the $N = 0, 1$ SLFTs is the spectrum of the degenerate representations [15–17] (see also [18,19]). We will show below that the $N = 2$ SLFT still possesses higher equations of motion despite these differences.

1. $N = 2$ SLFT

The $N = 2$ SLFT is based on the Lagrangian:

$$\mathcal{L} = \frac{1}{2\pi}\left(\partial\phi^-\bar\partial\phi^+ + \partial\phi^+\bar\partial\phi^- + \psi^-\bar\partial\psi^+ + \psi^+\bar\partial\psi^- + \bar\psi^-\partial\bar\psi^+ + \bar\psi^+\partial\bar\psi^-\right)$$
$$+ i\mu b^2\psi^-\bar\psi^- e^{b\phi^+} + i\mu b^2\psi^+\bar\psi^+ e^{b\phi^-} + \pi\mu^2 b^2 e^{b\phi^+ + b\phi^-} \tag{1}$$

* Corresponding author at: Department of Physics, Ewha Womans University, Seoul 120-750, South Korea.
E-mail addresses: ahn@ewha.ac.kr (C. Ahn), marian@inrne.bas.bg (M. Stanishkov), mstoilov@inrne.bas.bg (M. Stoilov).
[1] On leave of absence from INRNE, BAS, Bulgaria.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.11.057

where $(\phi^{\pm}, \psi^{\mp})$ are the components of a chiral $N=2$ supermultiplet, b is the coupling constant and μ is the cosmological constant. It is invariant under the $N=2$ superconformal algebra:

$$\begin{aligned}
&[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}\big(m^3 - m\big)\delta_{m+n},\\
&\big[L_m, G_r^{\pm}\big] = \left(\frac{m}{2} - r\right)G^{\pm}_{m+r}, \qquad \big[J_n, G_r^{\pm}\big] = \pm G^{\pm}_{n+r},\\
&\big\{G_r^+, G_s^-\big\} = 2L_{r+s} + (r-s)J_{r+s} + \frac{c}{3}\left(r^2 - \frac{1}{4}\right)\delta_{r+s},\\
&[L_m, J_n] = -nJ_{m+n}, \qquad [J_m, J_n] = \frac{c}{3}\delta_{m+n},
\end{aligned}\tag{2}$$

where $L_m, G_r^{\pm}$ and J_n are the modes of the corresponding conserved currents, the stress–energy tensor $T(z)$, the super-current $G(z)$ and the $U(1)$ current $J(z)$, and the central charge is:

$$c = 3 + \frac{6}{b^2}.$$

These are the left handed generators, there are in addition the right handed ones $\bar{L}_n$, $\bar{J}_n$, $\bar{G}_r^{\pm}$ closing the same algebra.

The basic objects are the primary fields (normal ordered exponents):

$$N_{\alpha,\bar{\alpha}} = e^{\alpha\phi^+ + \bar{\alpha}\phi^-},$$

the corresponding states being annihilated by the positive modes. These are the primary fields in the Neveu–Schwartz (NS) sector with r, s in (2) half-integer. There are in addition also Ramond (r, s — integer) primary fields $R_{\alpha,\bar{\alpha}}$ but we will not be concerned with them in this Letter. The conformal dimension and the $U(1)$ charge of the primary fields are:

$$\Delta_{\alpha,\bar{\alpha}} = -\alpha\bar{\alpha} + \frac{1}{2b}(\alpha + \bar{\alpha}), \qquad \omega = \frac{1}{b}(\alpha - \bar{\alpha}).\tag{3}$$

Among the primary fields there is a series of degenerate fields of the $N=2$ SLFT. They are characterized by the fact that at certain level of the corresponding conformal family a new primary field (i.e. annihilated by all positive modes) appears. Such fields can be divided in three classes (see e.g. [18]).

Class I degenerate fields are given by

$$\begin{aligned}
&N^{\omega}_{m,n} = N_{\alpha^{\omega}_{m,n}, \bar{\alpha}^{\omega}_{m,n}},\\
&\alpha^{\omega}_{m,n} = \frac{1-m}{2b} + (\omega - n)\frac{b}{2},\\
&\bar{\alpha}^{\omega}_{m,n} = \frac{1-m}{2b} - (\omega + n)\frac{b}{2}
\end{aligned}\tag{4}$$

m, n are positive integers. $N^{\omega}_{m,n}$ is degenerate at level mn and relative $U(1)$ charge zero. The irreducibility of the corresponding representations is assured by imposing the null-vector condition $D^{\omega}_{m,n}N^{\omega}_{m,n} = 0$, $\bar{D}^{\omega}_{m,n}N^{\omega}_{m,n} = 0$, where $D^{\omega}_{m,n}$ is a polynomial of the generators in (2) of degree mn and has $U(1)$ charge zero. It is normalized by choosing the coefficient in front of $(L_{-1})^{mn}$ to be 1. Let us give some examples of the corresponding null-operators:

$$\begin{aligned}
D^{\omega}_{1,1} &= L_{-1} - \frac{1}{2}b^2(1+\omega)J_{-1} + \frac{1}{\omega - 1}G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}},\\
D^{\omega}_{1,2} &= L^2_{-1} + b^2 L_{-2} - b^2(1+\omega)L_{-1}J_{-1} + \frac{b^2}{2}\big(1 + \omega - b^2(2+\omega)\big)J_{-2}\\
&\quad + \frac{b^4}{4}\omega(\omega+2)J^2_{-1} + \frac{2}{\omega - 2}L_{-1}G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}} - \frac{b^2\omega}{\omega - 2}J_{-1}G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}}\\
&\quad - \frac{b^2}{2}G^+_{-\frac{1}{2}}G^-_{-\frac{3}{2}} + \frac{b^2}{2}\frac{\omega+2}{\omega-2}G^+_{-\frac{3}{2}}G^-_{-\frac{1}{2}},\\
D^{\omega}_{2,1} &= L^2_{-1} + \frac{1}{b^2}L_{-2} - b^2(1+\omega)L_{-1}J_{-1} + \frac{1}{2}\big(b^2(1+\omega) - \omega - 2\big)J_{-2}\\
&\quad + \frac{1}{4}\big(b^4(\omega+1)^2 - 1\big)J^2_{-1} + \frac{2b^4\omega}{b^4(\omega-1)^2 - 1}L_{-1}G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}}\\
&\quad - \frac{b^2 + b^6(\omega^2 - 1)}{b^4(\omega - 1)^2 - 1}J_{-1}G^+_{-\frac{1}{2}}G^-_{-\frac{1}{2}} - \frac{b^4(\omega+1) + b^2 - 2}{2 + 2b^2(\omega - 1)}G^+_{-\frac{1}{2}}G^-_{-\frac{3}{2}}\\
&\quad + \frac{2 - b^2 + b^4(\omega - 1)(1 + b^2(\omega+1))}{2(b^4(\omega-1)^2 - 1)}G^+_{-\frac{3}{2}}G^-_{-\frac{1}{2}}.
\end{aligned}\tag{5}$$

The second class of degenerate fields is denoted by N^{ω}_m and comes in two subclasses IIA and IIB: