



Phantom-like behavior in $f(R)$ -gravity

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ABSTRACT

We investigate possible realization of the phantom-like behavior in the framework of $f(R)$ -gravity models where there are no phantom fields in the matter sector of the theory. By adopting some observationally reliable ansatz for $f(R)$, we show that it is possible to realize phantom-like behavior in $f(R)$ -gravity without introduction of phantom fields that suffer from instabilities and violation of the null energy condition. Depending on the choice of $f(R)$, the null energy condition is fulfilled in some subspaces of each model parameter space.

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1. Introduction

One of the most astonishing discoveries of the last two decades is the observation of a positively accelerated phase of cosmic expansion. This amazing result comes from several sources of observational data such as: measurements of luminosity-distances of supernovae type Ia (SNIa) [1], the cosmic microwave background (CMB) temperature anisotropies with the Wilkinson Microwave Anisotropy probe (WMAP) satellite [2], large scale structure [3], the integrated Sachs–Wolfe effect [4], and the weak lensing [5]. Indeed, general relativity with ordinary matter content of the universe leads to a decelerating universe and therefore it cannot describe this accelerating expansion which has been confirmed by a huge amounts of observational data. In order to realize this late-time acceleration theoretically, several approaches have been proposed. One possibility is to consider an extra source of energy–momentum with a negative pressure in the matter sector of the Einstein field equations. However, the nature of this extra component (the so-called dark energy) is yet unknown for cosmologists. A very simple and popular candidate for dark energy proposal is the cosmological constant [6], but this scenario suffers from some serious problems such as huge amount of fine-tuning and coincidence problems. Beside these problems, this scenario has not a dynamical behavior because of a constant equation of state parameter ($\omega_\Lambda = -1$). Another suggestion for dark energy is the dynamical models that include various scalar fields such as quintessence, k-essence, Chaplygin gas, phantom fields, quintom fields and so on [7]. On the other hand, one of the most important results of the observational data comes from WMAP5 that the equation of state parameter of dark energy can be less than -1 and even can have a transient behavior [8]. While general relativity with one scalar field cannot realize such a crossing behavior, non-minimal coupling of scalar field and gravity leads to this crossing phenomenon [9].

There is another approach to realize the cosmic speedup: modifying geometric part of the gravitational theory. This proposal can be realized in braneworld scenario (DGP model and its extensions [10]), string inspired scenarios (Gauss–Bonnet terms in the action [11]) and so on. A very popular modified gravity model is the so-called $f(R)$ -gravity [12] where $f(R)$ is an arbitrary function of the scalar curvature R . This scenario has the interesting feature that choosing an observationally reliable $f(R)$, it is possible to describe the early inflation as well as the late time acceleration of the universe in a fascinating manner [12]. Recently it has been shown that one can realize the phantom-like effect (increasing of the effective dark energy density with cosmic time and an equation of state parameter less than -1) in the normal branch of the DGP cosmological solution without introducing any phantom fields that violate the null energy condition (NEC) [13,14]. This type of the analysis then has been extended by several authors [15]. The main goal in these studies is the realization of the

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phantom-like behavior without introducing any phantom fields in the matter sector of the theory. In fact, since phantom fields suffer from instabilities and violate the null energy condition, it is desirable to realize this behavior without introduction of phantom fields. With this motivation, in this Letter we introduce another alternative to realize phantom-like effect: We study possible realization of this behavior in the framework of $f(R)$ -gravity models. We consider some observationally reliable versions of $f(R)$ -gravity and investigate the phantom-like behavior of each model without introducing any phantom field that violates the null energy condition. Some of these model such as Hu and Sawicki (HS) model have passed the solar system tests in a very good manner as well as the perturbation theory [16]. We show that all of these models in some subspaces of the model parameter space realize a phantom-like behavior without introducing any phantom fields. We study the conditions that are required in each case to fulfill the null energy condition.

2. $f(R)$ -gravity

In this section we consider the metric formalism of $f(R)$ -gravity and we summarize the field equations of the scenario. The action of a general $f(R)$ -gravity theory is given by [12,17–19]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \{ f(R) + \mathcal{L}_M \}, \quad (1)$$

where R is the scalar curvature, $f(R)$ is an arbitrary function of R and $\kappa = 8\pi G$ is the gravitational constant. The term \mathcal{L}_M accounts for the matter content of the universe. Using the metric approach, variation of this action with respect to $g_{\mu\nu}$ provides the field equation

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu f'(R) + g_{\mu\nu} \square f'(R) = \kappa T_{\mu\nu}^{(M)}, \quad (2)$$

where the prime denotes derivative with respect to R and the matter stress–energy density is defined as

$$T_{\mu\nu}^{(M)} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta(g^{\mu\nu})}. \quad (3)$$

By assuming a spatially flat FRW metric, the Friedmann equation can be written as

$$H^2 = \frac{8\pi G}{3} \left[\frac{\rho_M}{f'(R)} + \rho_{\text{curv}} \right], \quad (4)$$

where ρ_M is the energy density of the ordinary matter and ρ_{curv} is the energy density of the *curvature fluid* defined as

$$\rho_{\text{curv}} = \frac{1}{f'(R)} \left\{ \frac{1}{2} [f(R) - Rf'(R)] - 3H\dot{R}f''(R) \right\}. \quad (5)$$

Throughout this Letter we consider the Jordan frame, thus the continuity equation of the matter sector can be read as usual

$$\dot{\rho}_M = \rho_M(t = t_0) = 3H_0^2 \Omega_M (1+z)^3, \quad (6)$$

where Ω_M is the present day matter density parameter. The continuity equation for the curvature fluid is given in the following form [17]

$$\dot{\rho}_{\text{curv}} + 3H(1 + \omega_{\text{curv}}\rho_{\text{curv}}) = \frac{3H_0^2 \Omega_M \dot{R}f''(R)(1+z)^3}{[f'(R)]^2}. \quad (7)$$

By definition, the pressure of the curvature fluid is given by

$$P_{\text{curv}} = \frac{1}{f'(R)} \left\{ 2H\dot{R}f''(R) + \ddot{R}f''(R) + \dot{R}^2 f'''(R) - \frac{1}{2} [f(R) - Rf'(R)] \right\}. \quad (8)$$

The equation of state parameter corresponding to the curvature sector of the theory can be read as follows

$$\omega_{\text{curv}} = -1 + \frac{\ddot{R}f''(R) + \dot{R}[\dot{R}f'''(R) - Hf''(R)]}{\frac{1}{2}[f(R) - Rf'(R)] - 3H\dot{R}f''(R)}. \quad (9)$$

From the continuity equations (7) and field equation (4), the Hubble rate can be expressed as follows

$$\dot{H} = -\frac{1}{2f'(R)} \left\{ 3H_0^2 \Omega_M (1+z)^3 + \ddot{R}f''(R) + \dot{R}[\dot{R}f'''(R) - Hf''(R)] \right\}, \quad (10)$$

where $R = 6(\dot{H} + 2H^2)$. This equation is a very complicated equation and it is very difficult to solve it even with the simplest forms of the $f(R)$ -gravity.

3. Phantom-like behavior of $f(R)$ -gravity

With phantom-like behavior, we mean an effective energy density which is positive and grows with time and its equation of state parameter stays less than -1 . In this section, by adopting some cosmologically viable ansatz, we show that the modified gravity can lead to the effective phantom dark energy and phantom-like behavior without need to introduce any kind of the phantom (negative energy density) scalar fields that violate the null energy condition. To do this end, the modified Friedmann equation (4) can be expressed in a familiar form

$$H^2 = \frac{8\pi G_{\text{eff}}}{3} [\rho_M + f'(R)\rho_{\text{curv}}]. \quad (11)$$

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