Physics Letters B 680 (2009) 212–216

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On the dynamics of exotic matter: Towards creation of Perpetuum Mobile of third kind

Pavel Ivanov [a,b,*]

[a] Astro Space Centre of PN Lebedev Physical Institute, 84/32 Profsoyuznaya Street, Moscow 117810, Russia
[b] Department of Applied Mathematics and Theoretical Physics, CMS, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK

ARTICLE INFO

Article history:
Received 20 March 2009
Received in revised form 5 August 2009
Accepted 27 August 2009
Available online 3 September 2009
Editor: S. Dodelson

PACS:
03.30.+p
04.20.-q
47.75.+f
95.36.+x

Keywords:
Hydrodynamics
Relativity
Cosmology
Dark energy

ABSTRACT

The one-dimensional dynamics of a classical ideal 'exotic' fluid with equation of state $p = p(\epsilon) < 0$ violating the weak energy condition is discussed. Under certain assumptions it is shown that the well-known Hwa–Bjorken exact solution of one-dimensional relativistic hydrodynamics is confined within the future/past light cone. It is also demonstrated that the total energy of such a solution is equal to zero and that there are regions within the light cone with negative $(-)$ and positive $(+)$ total energies. For certain equations of state there is a continuous energy transfer from the $(-)$-regions to the $(+)$-regions resulting in indefinite growth of energy in the $(+)$-regions with time, which may be interpreted as action of a specific 'Perpetuum Mobile' (Perpetuum Motion). It is speculated that if it is possible to construct a three-dimensional non-stationary flow of an exotic fluid having a finite negative value of energy such a situation would also occur. Such a flow may continuously transfer positive energy to gravitational waves, resulting in a runaway. It is conjectured that theories plagued by such solutions should be discarded as inherently unstable.

1. Introduction

If physical laws do not prohibit the presence of exotic matter violating the weak energy condition[1] and having some other certain properties many exciting possibilities arise. For example, solutions of the Einstein equations coupled to an exotic matter include wormholes, time machines (e.g. [1]) and cosmological models with energy density of the Universe growing with time (see, e.g. [2] for a review and discussion of cosmological consequences) leading to the so-called cosmological Doomsday, see e.g. [3]. Since theories incorporating an exotic matter may lead to counterintuitive and, possibly, physically inconsistent effects it appears to be important to invoke different thought experiments, which could clarify self-consistency of such theories. Here we discuss such an experiment and explicitly show that in a class of models containing an exotic matter of a certain kind there could be ever expanding with time separated regions of space having positive and negative total energies and that absolute values of the energies in these regions could grow indefinitely with time while the energy of the whole physical system is conserved. This is based on the property of the exotic matter to have negative energy density measured by observers being at rest with respect to some Lorentz frame and, accordingly, given by the (tt)-component of the stress–energy tensor, T^{tt}, provided that there are sufficiently large fluid velocities with respect to this frame. We also speculate that in a more advanced variant of our model there could be an isolated region of space filled by an exotic matter with its total energy indefinitely decreasing with time due to processes of interaction with some other conventional physical fields carrying positive energy. One of such processes could be emission of gravitational waves. If conditions for emission of gravitational waves are always fulfilled in the course of evolution positive energy is continually carried away from the region, which results in a runaway. In this respect it is appropriate to mention that the known results on positiveness of mass in General Relativity are not valid for the matter violating the weak energy condition, see e.g. [4] and the energy of an isolated region could evolve from positive to negative values. Although such a situation resembles the action of a Perpetuum Mobile of second kind, where heat transfer from a

* Address for correspondence: Astro Space Centre of PN Lebedev Physical Institute, 84/32 Profsoyuznaya Street, Moscow 117810, Russia.
E-mail address: pbi20@cam.ac.uk.

[1] Let us remind that the weak energy condition is said to be violated for a matter having the stress–energy tensor $T_{\mu\nu}$ if there is such a time-like future directed vector field t^{μ} that the inequality $T_{\mu\nu}t^{\mu}t^{\nu} < 0$ holds in some region of space–time.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2009.08.057

colder part of an isolated system to a hotter part occurs, the notion of 'temperature' looks ambiguous in our case and, therefore, because of the lack of notation, we refer to this hypothetical effect as a 'Perpetuum Mobile of third kind'. In many applications a phenomenological description of the exotic matter assumes that the matter dynamics can be described by a hydrodynamical model with the stress–energy tensor of an ideal Pascal fluid, which can be fully specified by its equation of state, $p = p(\epsilon)$, where p is the pressure and ϵ is the comoving energy density, which is assumed to be positive below. In this case, violation of the weak energy condition can be reformulated as a requirement that the pressure is negative with its modulus exceeding the value of the comoving energy density. We deal hereafter only with simplest models of hydrodynamical type and neglect the effects of General Relativity and interactions with other physical fields. Therefore, in the model explicitly considered in the text the transfer of energy from one region of space to another, in particular, between the regions having total energies of different signs, is provided by hydrodynamical effects. However, as we have mentioned above, it seems reasonable to suppose that an analogous runaway effect may happen in a more realistic situation, where e.g. energy is continually carried away from a spacial region having negative total energy by gravitational waves.[2] Taking into account that gravitational interaction is universal, the proposed 'Perpetuum Mobile of third kind' may represent a difficulty in theories, where it emerges. We believe that such theories are inherently unstable and must, therefore, be discarded.

It is important to note, however, that in a general scenario the region emitting gravitational waves could have a non-linear three-dimensional dynamics. Explicit solutions of this kind may be quite difficult to obtain due to severe technical problems.

2. Basic definitions and equations

Let us discuss a one-dimensional planar relativistic flow of an ideal fluid with a baratropic equation of state

$$p = p(\epsilon), \tag{1}$$

where p is the pressure and ϵ is the comoving energy density. As has been mentioned in Introduction, we are going to consider later in the text the case of an exotic fluid, where the pressure is negative and the weak energy condition is violated. For a baratropic fluid this leads to:

$$\sigma \equiv -p > \epsilon, \tag{2}$$

where we introduce the negative of pressure p, $\sigma = -p$. Since only one-dimensional flows will be considered, we can also apply our analysis to a situation, where a fluid has an anisotropic stress tensor. Say, we can assume only one of its components to be non-negative and proportional to $\delta(y, z)$, where y and z are the Minkowski spacial coordinates corresponding to directions perpendicular to the direction of motion. In this case, equations of motion will describe dynamics of a straight string consisting of exotic matter.

Equations of motion may be written in a divergent form reflecting the laws of conservation of energy and momentum

$$T^{tt}_{,t} + T^{tx}_{,x} = 0, \qquad T^{tx}_{,t} + T^{xx}_{,x} = 0, \tag{3}$$

where (t, x) are the standard Minkowski coordinates, comma stands for differentiation, and T^{ij} are the corresponding components of the stress–energy tensor:

$$T^{tt} = \gamma^2\left(\epsilon + v^2 p\right), \qquad T^{tx} = \gamma^2 v(\epsilon + p),$$
$$T^{xx} = \gamma^2\left(p + v^2\epsilon\right), \tag{4}$$

where v is the three-velocity and $\gamma = \frac{1}{\sqrt{1-v^2}}$.

3. The Hwa–Bjorken solution and the Milne coordinates

As has been first shown by Hwa [8] and later by Bjorken [9], the set of Eqs. (3) has an especially simple 'acceleration-free' solution valid for a fluid having an arbitrary equation of state. In this solution velocity of any fluid element conserves along the path of the fluid element and the velocity field has a very simple form

$$v = x/t \equiv \xi. \tag{5}$$

For a baratropic fluid the distribution of energy is given by another simple implicit relation

$$\tau = \exp\left\{-\int_{\epsilon_*}^{\epsilon} \frac{d\epsilon'}{\epsilon' + p(\epsilon')}\right\} = \exp\left\{-\int_{\epsilon}^{\epsilon_*} \frac{d\epsilon'}{\sigma(\epsilon') - \epsilon'}\right\}, \tag{6}$$

where

$$\tau = \sqrt{t^2 - x^2}, \tag{7}$$

and ϵ_* is a constant of integration.

Obviously, Eqs. (6)–(7) are defined only inside the future/past light cone, $|t| > |x|$, in an effective two-dimensional Minkowski space described by the metric

$$ds^2 = dt^2 - dx^2. \tag{8}$$

The analytic continuation of the solution on the right/left Rindler wedge $|t| < |x|$ is straightforward.

Although the two-dimensional Minkowski space appears naturally due to one-dimensional character of the problem let us remind that the problem is defined in four-dimensional Minkowski space. Therefore, for the problem with Pascal pressure, where it is assumed that all variables do not depend on the coordinates (y, z) perpendicular to x, it is better to say that from the four-dimensional point of view the condition $|t| = |x|$ determines four-dimensional "light wedges" since it does not depend on directions perpendicular to the direction of motion.

The energy density ϵ is equal to zero on the light cone $|t| = |x|$ if and only if the integrals in the exponents in Eq. (6) are positive and diverge when $\epsilon \to 0$. Accordingly, in this case, the condition (2) must be satisfied. Additionally, in order to make the integrals divergent we must have

$$\sigma - \epsilon \leqslant \left|O(\epsilon)\right| \tag{9}$$

when $\epsilon \to 0$. Provided that condition (9) is valid the solution may be considered as confined within the future/past light cone with no flows of energy and momentum through the cone boundary. For

---

[2] Note that one should distinguish between the standard hydrodynamical instabilities e.g. the ones operating in a fluid having a negative square of speed of sound and the instability related to violation of the weak energy condition. While the former could lead to a highly irregular non-linear dynamics of the system they cannot themselves result in formation of spacial regions having a negative total energy, and, a runaway of the kind discussed in this Letter, see also the footnote in the next section.

Let us also stress that the runaway effect is different from the well-known instabilities of linear modes of stationary hydrodynamical flows having a negative energy difference with respect to a stationary configuration like the Chandrasekhar–Friedman–Schutz instability [5,6] or the instability operating in shear flows, see e.g. [7] and references therein. Although the linear instabilities can lead to a decrease of energy of the main flow they cannot bring a system non-violating the weak energy condition to a state with a negative total energy. Therefore, in the latter case the runaway effect of the type we consider in this Letter is not possible.