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Emergence of symmetries

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ABSTRACT

The mechanism of symmetry formation is discussed in the framework of multidimensional gravity. It is shown that this process is strictly connected to the entropy decrease of compact space. The existence of low energy symmetries is not postulated from the beginning. They could be absent during the inflationary stage under certain conditions discussed in the Letter.

1. Introduction

The idea of multidimensional space–time allows to clarify some fundamental questions, such as the problems of modern cosmology and the Standard Model which are discussed in terms of extra-dimensional gravity [1–7]. As was shown in [8], the numerical values of the fundamental parameters depend on geometry of extra dimensions. The existence of gauge symmetries may be related to isometries of extra space [8–10]. Hereinafter we consider compact d-dimensional extra spaces in the spirit of the Universal Extra Dimensions (UED). It is supposed that symmetries of extra space are related to the low energy symmetries in the observable Universe. The size of extra dimensions varies in the wide range from the Planck scale up to 10^{-18} cm, the upper limit known from the particle physics. This approach is similar to the UED one though no fields except metric tensor are considered. An applicability of the results discussed below to another approaches like ADD [1] and Randall–Sundrum models [3] is the subject of future investigations.

(Maximally) symmetric metrics of extra space as a starting point are among the most popular in the literature, see e.g. [10–12]. This assumption makes it possible to obtain clear and valuable results. At the same time we must take into account the quantum origin of space itself due to fluctuations in the space–time foam. There is no reason to assume that the geometry or/and topology of extra space is simple [13,14]. Moreover it seems obvious that a measure $\mathfrak{M}$ of all symmetrical spaces equals zero so that the probability of their nucleation $\mathcal{P} = 0$. Hence some period of extra space symmetrization ought to exist [8,15–18].

In the present Letter we investigate the entropic mechanism of space symmetrization after its nucleation. It is shown that the stabilization of the extra space and its symmetrization are proceeding simultaneously. This process is accompanied by a decrease in entropy for the extra space and an increase in entropy for main one.

2. Time dependence of compact space geometry

As was mentioned above some mechanism of the extra space symmetrization should exist. In this section we consider some toy models to clarify the situation.

As a common basis, consider a Riemannian manifold

$$T \times M \times M' \tag{1}$$

with the metric

$$\begin{aligned} ds^2 &= G_{AB}\, dX^A\, dX^B \\ &= dt^2 - g_{mn}(t,x)\, dx^m\, dx^n - \gamma_{ab}(t,x,y)\, dy^a\, dy^b. \end{aligned} \tag{2}$$

Here M, M' are the manifolds with spacelike metrics $g_{mn}(t,x)$ and $\gamma_{ab}(t,x,y)$ respectively, T denotes the timelike direction. The set of coordinates of the subspaces M is denoted by x; y is the same for M'. We will refer to M and M' as a main space and a compact extra space respectively. The curvature of the manifold is assumed to be arbitrary.

Firstly, consider the $(d+1)$-dimensional compact manifold $M' \times T$ with metric

$$ds^2 = dt^2 - \gamma_{ab}(t,y)\, dy^a\, dy^b, \qquad \gamma_{ab}(y,t) = \eta_{ab} + h_{ab}(t,y).$$

For the Einstein–Hilbert action

$$S = \int d^d y\, dt\, \sqrt{|\gamma|} R \tag{3}$$

and in the limit $h_{ab} \ll 1$, classical equations have the form [9]

$$\Box_{d+1} h_{ab} = 0, \tag{4}$$

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where

$$\Box_{d+1} \equiv \frac{1}{\sqrt{\gamma}}\partial_0(\sqrt{\gamma}\partial_0) + \frac{1}{\sqrt{\gamma}}\partial_a\big(\sqrt{\gamma}\gamma^{ab}\partial_b\big). \tag{5}$$

This wave equation has no static symmetrical solutions if initial conditions are arbitrary. This would mean the absence of symmetries even in the modern epoch, which is unacceptable.

The situation changes considerably if we take into account the dynamics of the main manifold M. Let it possess the Friedmann–Robertson–Walker (FRW) metric and the scale factor $a(t)$ (we assume $\dot{a}(t) > 0$). If our Universe plays the role of the manifold M the latter statement is just the observable fact. The equation of motion for the metrics of the extra space M' acquires the form

$$\Box_{d+1} h_{ab} + 3H\dot{h}_{ab} = 0, \tag{6}$$

where the Hubble parameter $H = \dot{a}/a > 0$. We also took into account the form of metrics (2) and equality

$$\frac{1}{\sqrt{g}}\partial_0\sqrt{g} = 3\frac{\dot{a}}{a} = 3H > 0 \tag{7}$$

valid for 4-dimensional FRW space. The term $3H\dot{h}_{ab}$ in (6) indicates the presence of friction and gives the asymptotic $\gamma_{ab} \to \text{const}$ for $t \to +\infty$.

So the dynamics of the main space M could cause the stabilization of the extra space M'. Note that friction usually means entropy increasing in any system.

As a more complex and valuable example consider a gravity with higher order derivatives and the action in the form

$$S = \int d^{D+1}z\sqrt{G}f(R), \tag{8}$$

where $z = (t, x, y)$ and $G = |\det g \cdot \det \gamma|$. The metric of extra space (2) is chosen in the form $\gamma_{ab} = \gamma_{ab}(t, y)$. We also use inequality

$$R_M \ll R_{M'} \tag{9}$$

for the Ricci scalar of the main space R_M and the Ricci scalar of the extra space $R_{M'}$. It is known that in the framework of D-dimensional gravity linear in the Ricci scalar the stabilization of an extra space is impossible without involving additional fields [16,19,20]. On the other side a D-dimensional gravity with higher derivatives gives such an opportunity. The process of stabilization of the extra space is discussed in the papers [18,21] where it was shown that a stationary size of extra space depends on its dimensionality and initial parameters of the pure gravitational Lagrangian.

Recall that action (8) is equivalent to linear action with an additional scalar field [21–23]

$$S = \int d^{D+1}z\sqrt{\tilde{G}}\big[\tilde{R}(\tilde{G}) + \tilde{G}^{ab}\partial_a\phi\partial_b\phi - 2U(\phi)\big], \tag{10}$$

where

$$\phi = \frac{1}{A}\ln f'(R), \quad A = \sqrt{\frac{D-1}{D}}, \tag{11}$$

$$U(\phi) = \frac{1}{2}e^{-B\phi}\big[R(\phi)e^{A\phi} - f\big(R(\phi)\big)\big], \quad B = \frac{D+1}{\sqrt{(D-1)D}}. \tag{12}$$

The details can be found in the papers cited above. The classical equation of motion of action (10) has the form

$$\ddot{\phi} + 3H\dot{\phi} + \Box_{d+1}\phi + U'(\phi) = 0, \tag{13}$$

where metric (2), equality (7) and definition (5) are taken into account. As in the previous cases, the term containing the Hubble parameter H is responsible for friction in the system.

Let the potential (12) has a minimum at ϕ_m. Due to the presence of the friction, the additional scalar field ϕ tends to a constant. According to (11) the Ricci scalar of the extra space is connected to the scalar field and also tends to a stationary value,

$$R \to R(\phi_m). \tag{14}$$

The observed main space is described by the FRW metric and its Ricci scalar tends to zero, $R_M \sim 1/a(t \to \infty)^2 \to 0$ so that we may neglect its contribution at large times. Thus the extra space M' acquires a maximally symmetrical form.

As in the previous case, see (6), it is the dynamics of the main space that is responsible for the friction in the extra space and its stabilization. This indicates the presence of entropy flow from the extra space M' to the main one M [24,25].

3. Entropy and symmetry formation

In the previous section we saw that stabilization of extra space, an extension of its symmetry group and an entropy increase in a whole space proceed simultaneously. Below we show that the entropy of a compact extra space is decreasing with time. To this end we prove the following

Statement. *Let M be a smooth manifold, G_1 and G_2 are two given metrics on it. If the number of Killing vectors of metric G_1 is less then the number of Killing vectors of metric G_2 then the entropy of G_1 is greater than the entropy of G_2.*

Proof. We will use the well-known definition of the Boltzmann entropy S. It links entropy to a number of microstates Ω, $S = k_B \ln \Omega$. Other definitions are discussed in [26,27] for example.

Let us consider a compact smooth manifold M. We suppose that every metric G on M defines a microstate. More definitely, two metrics G_1 and G_2 on M define the same microstate if and only if they are equal in each of the points $P \in M$.

The definition of a macrostate is as follows. Let v be an arbitrary smooth vector field defined globally on the smooth manifold M. Any shift along the integral path of vector field v corresponds to a diffeomorphism M on itself. We define a macrostate as a set of metrics G that are connected by shifts. As an example, a 2-dim torus with a bulge, being shifted, still represents the same macrostate. Another macrostate is determined by the addition of another bulge. So this definition seems reasonable.

The statistical weight of a given macrostate is the number of microstates. The latter is a continuum set for any classical system. The concept of microstates is correctly defined at a quantum level where the set of energy levels is known. However, the quantization of geometry is a yet unsolved problem. That is why any discussion of a metric on scale less than the Planck scale L_P is pointless. Thus shifts less than Planck scale should not be taken into account when counting statistical weight (see discussion in [28]). Therefore a number of shifts along various integral paths is assumed to be finite.

Let us compare statistical weights of two metrics G_1 and G_2 with the same number of shifts at manifold M. Let G_1 have no Killing vectors and G_2 possesses a global Killing field. Shifts along Killing vector of G_2 lead to the same microstate by definition. So the statistical weight of G_1 is greater than the statistical weight of G_2. A similar argument is correct in the general case as well when the number of Killing vectors of metrics G_1 is less then the number of Killing vectors of metrics G_2. This statement is also valid for the entropy which is the nondecreasing function of the statistical weight. Therefore