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Scattering states of Woods–Saxon interaction in minimal length quantum mechanics

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ABSTRACT

We propose a very simple approach to deal with the problems of the modified Schrödinger equation due to minimal length and thereby solve the minimal length Schrödinger equation in the presence of a non-minimal Woods–Saxon interaction. The transmission and reflection coefficients are reported as well.

1. Introduction

There are many evidences for a minimal length (ML) of order of Planck length $l_P = \sqrt{G\hbar/c^3} \approx 10^{-35}$ m [1]. In particular, quantum gravity, string theory and black-hole physics are among such cases. Appearance of a minimal length modifies the traditional Heisenberg uncertainty principle into the form which we call the generalized uncertainty principle (GUP for short) in the jargon. On the other hand, such a deformed uncertainty relation changes the corresponding wave equation and as a result the physics of the system. The corresponding Schrödinger equation is now no more a second-order differential equation and this is the situation that causes a great difficulty. Some authors have investigated the problem and the consequences in connection with string theory [1], quantum gravity [2], quantum groups [3], algebraic structure [4], Hilbert space representation [5], black-hole physics [6], path integral approach [7], relativity [8], extra-dimensions [9] and the holograph [10], non-tachyonic bosonic string [11], doubly special relativity [12], Big-Bang singularity [13], WKB approximation [14], cosmological constant [15], relativistic wave equations [16], Lamb shift, Landau levels and the tunneling current [17,18]. Here, we first revisit the modified quantum mechanics due to minimal length. More precisely, in previously published papers we are left with a six-order ordinary differential equation which is too cumbersome to be simply solved via the common techniques of mathematical physics. To get rid of such complexity, some authors ignore the six-order derivative and they are therefore left with a fourth-order one which is of course easier than the former. But, the story can be much simpler via the analytical techniques already known for decades if we change the space of the problem via replacing the momentum term with its alternative, i.e. $p^4 = 4m^2(E_n^{(0)} - V(x))^2$ and $p^2 = 2m(E_n^{(0)} - V(x))$. Such a replacement leaves us with neither a six-order nor a fourth-order differential equation. Rather, we are left with an ordinary differential equation with a different effective potential. We next apply our idea to the Woods–Saxon (WS) potential [19] which is a successful phenomenological potential in nuclear and particle physics.

2. The generalized uncertainty principle

An immediate consequence of the ML is the GUP [1,12]

$$\Delta x \geqslant \frac{\hbar}{\Delta p} + \alpha l_p^2 \frac{\Delta p}{\hbar}, \tag{1}$$

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where the GUP parameter α is determined from a fundamental theory [1,12]. At low energies, i.e. energies much smaller than the Planck mass, the second term in the right hand side of Eq. (1) vanishes and we recover the well-known Heisenberg uncertainty principle. The GUP of Eq. (1) corresponds to the generalized commutation relation [12]

$$[x_{op}, p_{op}] = i\hbar\big(1+\beta p^2\big), \quad 0 \leqslant \beta \leqslant 1 \tag{2}$$

where $x_{op} = x$, $p_{op} = p[1+\beta(p)^2]$ and $0 \leqslant \beta \leqslant 1$. The limits $\beta \to 0$ and $\beta \to 1$ correspond to the standard quantum mechanics and extreme quantum gravity, respectively. Eq. (2) gives the minimal length in this case as $(\Delta x)_{\min} = 2l_p\sqrt{\alpha}$.

3. A different methodology

Let us first recall few points; the WS potential has the form

$$V(x) = V_0\left[\theta(-x)\frac{1}{1+qe^{\frac{-x-R}{a}}} + \theta(x)\frac{1}{1+\tilde{q}e^{\frac{x-R}{a}}}\right], \tag{3}$$

where V_0, q, a and R represent the potential depth, deformation parameter, surface thickness and nuclear radius, respectively. The modified Schrödinger equation for a free particle is (for a detailed discussion see [5,6])

$$\left\{\frac{p_{op}^2}{2m} + V(x)\right\}\psi_n(x) = E_n\psi_n(x). \tag{4}$$

On the other hand,

$$\begin{aligned} p^2 &= 2m\big(E_n^{(0)} - V(x)\big),\\ p^4 &= 4m^2\big(E_n^{(0)} - V(x)\big)^2, \end{aligned} \tag{5}$$

where $E_n^{(0)}$ is the eigenvalue of

$$H_0 = \frac{p^2}{2m} + V(x). \tag{6}$$

Thus, from Eqs. (2)–(6), we may write

$$H = \frac{p^2}{2m} + \frac{\beta p^4}{m} + \frac{\beta^2 p^6}{2m} + V(x) = H_0 + \frac{\beta p^4}{m} = H_0 + 4m\beta\big(E_n^{(0)} - V(x)\big)^2, \tag{7}$$

or, in differential form,

$$\frac{\hbar^2}{2m}\frac{d^2\psi_n(x)}{dx^2} + \big(E_n - 4m\beta\big(E_n^{(0)} - V(x)\big)^2 - V(x)\big)\psi_n(x) = 0. \tag{8}$$

4. Reflection and transmission coefficients

As we are searching for the scattering states of the equation for a WS potential barrier, first we will study the wave functions for $x < 0$. From substitution of Eq. (3) into Eq. (8), we find

$$\frac{\hbar^2}{2m}\frac{d^2\psi_L(x)}{dx^2} + \left\{E_n - 4m\beta\big(E_n^{(0)}\big)^2 - \frac{4m\beta V_0^2}{(1+qe^{\frac{-x-R}{a}})^2} + \frac{8m\beta E_n^{(0)}V_0}{1+qe^{\frac{-x-R}{a}}} - \frac{V_0}{1+qe^{\frac{-x-R}{a}}}\right\}\psi_L(x) = 0. \tag{9}$$

By considering $y_L = -x - R$, $\alpha = 1/a$, we have

$$\frac{\hbar^2}{2m}\frac{d^2\psi_L(y_L)}{dy_L^2} + \left\{E_n - 4m\beta\big(E_n^{(0)}\big)^2 - \frac{4m\beta V_0^2}{(1+qe^{\alpha y_L})^2} + \frac{8m\beta E_n^{(0)}V_0}{1+qe^{\alpha y_L}} - \frac{V_0}{1+qe^{\alpha y_L}}\right\}\psi_L(y_L) = 0. \tag{10}$$

Applying the new variable $z_L = \frac{1}{1+qe^{\alpha y_L}}$ the latter is written as

$$z_L(1-z_L)\frac{d^2\psi_L(z_L)}{dz_L^2} + (1-2z_L)\frac{d\psi_L(z_L)}{dz_L} + \frac{1}{z_L(1-z_L)}\big\{M_L z_L^2 + N_L z_L + P_L\big\}\psi_L(z_L) = 0, \tag{11}$$

where

$$\begin{aligned} M_L &= -\frac{8m^2\beta V_0^2}{\hbar^2\alpha^2},\\ N_L &= \frac{16m^2\beta V_0 E_n^{(0)}}{\hbar^2\alpha^2} - \frac{2V_0 m}{\hbar^2\alpha^2},\\ P_L &= \frac{2E_n m}{\hbar^2\alpha^2} - \frac{8m^2\beta(E_n^{(0)})^2}{\hbar^2\alpha^2}. \end{aligned} \tag{12}$$