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Topological fluctuations in dense matter with two colors

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ABSTRACT

We study the topological charge fluctuations of an SU(2) lattice gauge theory containing both $N_f = 2$ and 4 flavors of Wilson fermion, at low temperature with non-zero chemical potential μ. The topological susceptibility, χ_T, is used to characterise differing physical regimes as μ is varied between the onset of matter at μ_o and color deconfinement at μ_d. Suppression of instantons by matter via Debye screening is also investigated, revealing effects not captured by perturbative predictions. In particular, the breaking of scale invariance leads to the mean instanton size $\bar{\rho}$ becoming μ-dependent in the regime between onset and deconfinement, with a scaling $\bar{\rho} \propto \mu^{-2}$ over the range $\mu_o < \mu < \mu_d$, resulting in an enhancement of χ_T immediately above onset.

1. Introduction

Lattice studies of matter at non-zero baryon density are hampered by the 'sign problem', which arises when a quark chemical potential term μ is included in the Euclidean QCD action. The resulting complex nature of the fermion determinant precludes a positive definite probability measure and computational techniques based on importance sampling break down. A gauge theory which is accessible to Monte Carlo simulations is QC_2D, based on gauge group SU(2), describing "two color matter". In QC_2D, quarks belong in the pseudoreal **2** representation of SU(2) which can guarantee a positive definite measure.

Studies of two color matter have been performed utilising a number of fermion formulations. The series of works obtained from simulations involving two and four flavors of Wilson fermion [1–3] have revealed a scenario in which, as μ is increased, baryonic matter forms at an onset $\mu_o = m_\pi/2$ whereupon the matter then exists in a superfluid state with a progression from a dilute gas of tightly-bound diquark pairs to degenerate quark matter, culminating in color deconfinement at around $\mu \approx 1.1 m_\pi$. This Letter supplements this picture with an investigation of topological effects observed on the same lattice configurations.

The topological charge density q_T may be defined in terms of the Yang–Mills field tensor as

$$q_T = \frac{1}{32\pi^2} F_{\mu\nu}\tilde{F}_{\mu\nu} \tag{1}$$

with $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$. The action is minimised when the condition $F_{\mu\nu} = \pm\tilde{F}_{\mu\nu}$ is satisfied. The observable measured to study topological charge fluctuations is the *topological susceptibility*, χ_T, defined as

$$\chi_T = \frac{\langle Q^2\rangle}{V}, \tag{2}$$

where $Q = \int d^4x\, q_T$ and $V = \int d^4x$. Using large-N_c methods χ_T is estimated by means of the Witten–Veneziano formula [4,5]

$$\chi_T = \frac{f_\pi^2}{2N_f}\left(m_\pi^2 + m_{\eta'}^2 - 2m_K^2\right) \tag{3}$$

to be (180 MeV)^4 in the SU(3) gauge vacuum. Simulations of hot two color matter with two flavors of staggered quark (equivalent to $N_f = 8$ continuum quark flavors) have shown this quantity drops sharply at the deconfining temperature and have suggested this also happens at non-zero chemical potential [6,7]. When χ_T is measured as a function of $a\mu$, the susceptibility remains constant before dropping dramatically at a critical chemical potential corresponding to both deconfinement and chiral symmetry restoration.

In a semi-classical picture topological charge is localised on four-dimensional objects called instantons, which are solutions of the self-dual condition for a local minimum of the action [8]. Another observable of interest is the size of an instanton ρ. This is a measure of the extent to which the gauge field action is localised. For classical Yang–Mills instantons the size may be considered arbitrary due to scale invariance and so ρ does not depend upon the action, and vice versa. However, in the quantum vacuum scale invariance is broken, and the typical size of an instanton is estimated to be in the region of 0.3 fm [9,10].

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In dense matter, Debye screening of color charge leads to instanton suppression [11]. Perturbative calculations [12] predict that instanton number at large chemical potential should go like

$$n(\mu) = n(\mu = 0)\exp\big(-N_f(\rho\mu)^2\big). \tag{4}$$

Therefore, as the number of quark flavors N_f is increased, instantons should be suppressed and χ_T should decrease. It should also be expected that, if the average instanton size ρ is indeed fixed, then the extra matter present as μ is increased will screen the topological charge and suppress χ_T still further.

2. Methodology

In order to explore instanton effects on a lattice we replace the continuum topological charge density q_T (1) with its lattice counterpart

$$q_L(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}\big(U_{\mu\nu}(x)U_{\rho\sigma}(x)\big) \tag{5}$$

where $U_{\mu\nu}(x)$ is the product of link variables around a plaquette at site x in the μ–ν plane [13]. The charge density is thus measured by taking the trace of the product of two orthogonal plaquettes. The total charge Q_L is obtained via $Q_L = \sum_x q_L(x)$. Within each configuration, the peaks due to the presence of instantons (whose structure may extend over a scale $\rho \gg a$, where a is the lattice spacing) are mutated by short scale ($\mathcal{O}(a)$) fluctuations. Such UV fluctuations are highly undesirable as they contribute to the total charge but obscure the 'real' instantons, and so the measured susceptibility can be an overestimate [14]. The lattice topological susceptibility $\chi_L \equiv \langle Q_L^2\rangle/V$ differs from the continuum value by both a multiplicative factor Z and an additive one M:

$$\chi_L = Z^2 a^4 \chi_T + M. \tag{6}$$

Z and M depend on several factors including the quark mass, the inverse coupling β and the choice of fermion operator [7]. In general, on the lattice, $Z \neq 1$ and the charge Q_L is not integer-valued. The challenge is to minimise the unwanted, short distance contributions while in the process recovering the continuum value in an unambiguous fashion.

Q_L for a given configuration of gauge fields is calculated by means of Eq. (5). The effects of UV fluctuations are minimised by cooling [15], whereby a new configuration is generated from the old by visiting lattice sites in turn and minimising the action locally. Repeating this successively has the effect of smoothing out fluctuations and revealing the underlying topological structure in the gauge fields. By prudent use of cooling, the multiplicative factor $Z \to 1$ as the unwanted fluctuations are eliminated. However, excessive cooling eliminates not just the UV fluctuations but will also shrink and ultimately eradicate the 'real' instantons. If cooling shrinks an instanton until its size $\rho < a$ then it 'falls through' the lattice and some of the topological information is lost. If only larger instantons contribute to the total charge then there is a tendency to underestimate Q_T. Information can also be lost as too much cooling has a tendency to annihilate instanton–anti-instanton pairs. The total charge may remain the same but the charge density is reduced. Therefore, it is vital that good control of the cooling process is maintained.

The additive constant M may be dealt with by equating it to the value of the topological susceptibility in the $Q_T = 0$ sector, setting $M = \chi_0 \equiv \chi_T(Q = 0)$. As we have no prior knowledge to suggest that our ensemble is in the trivial sector we must modify Eq. (2). In the non-trivial sector M can be eradicated by redefining

$$a^4\chi_T = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}. \tag{7}$$

Thus, by measuring the charges on a number of cooled field configurations with $Z \sim 1$ and calculating χ_T by means of (7), the physical topological susceptibility can be extracted from the lattice one. Henceforth, we discard the references to lattice values via our subscripts L and merely label χ and Q with the subscript T.

The cooling method employed here uses a computer program to read the gauge field information from each configuration and then calculate the total action by summing over the plaquettes. In general, this is not the minimum action. A point is then chosen and a link variable $U_\mu(x)$ is selected. There are 6 plaquettes with this link in common. The code sums the link products, in the form of unitary matrices which form the 'staples' bordering the link $U_\mu(x)$, resulting in a 2×2 matrix V. The matrix V is non-unitary and must be renormalised as $\tilde{V} = (\mathrm{Det}\,V)^{-1/2}V$. Keeping $\tilde{V}$ fixed, the action is then minimised by modifying $U_\mu(x)$. By systematically working through the old configuration and updating all links $U_\mu(x)$ a new configuration is produced with a lower action than the original one. This completes the first cooling sweep. By predetermining the number of sweeps to be performed, the process repeats automatically and the configuration is cooled to the required extent. When cooling is complete, the code then searches through the final configuration to find where the peaks of the action are located and $F\tilde{F}$ at these points is recorded. Setting a minimum cutoff for $F\tilde{F}$ allows the code to disregard the smallest fluctuations. Imposing a second cutoff for the maximum extent of the gauge fields inside an instanton minimises any finite volume effects associated with excessively large instantons. Once the required topological information is extracted from the cooled configuration, the program then moves onto the next configuration in the ensemble and repeats as necessary.

To find the total topological charge on each configuration, a second program obtains the net value of all the peaks of $F\tilde{F}$ from the output of the first, providing a sequence of estimates for the fluctuating variable Q_T. The topological susceptibility is estimated from this using Eq. (7).

One aspect of topological structure that is worth investigating is the size distribution of the instantons. Instanton size may be calculated from the peak value of the topological charge density using

$$q_{\mathrm{peak}} = \frac{6}{\pi^2\rho^4}. \tag{8}$$

This classical approximation works reasonably well for large lattice instantons, but for smaller ones whose size is of the order of the lattice spacing, corrections of $\mathcal{O}(a^2)$ are needed. The necessary correction factors for $N_c = 3$ were calculated by Smith and Teper by cooling a classical instanton and then parametrising the resulting relationship between Q and ρ [14]. The computational method employed in this study involved reading the peak values of the charge from the lattice configurations and then applying iterative bisection to find a value for ρ which satisfied Eq. (8) to within a predetermined error factor ϵ.

3. Numerical results

Information about the topological structure was extracted using two different gauge field ensembles. The first was generated on a $12^3 \times 24$ lattice with $\sqrt{\sigma}a = 0.415(18)$ (σ is the string tension) using $N_f = 2$ flavors of Wilson fermion at an inverse coupling $\beta = 1.9$ [2]. The fermion action included a diquark source term $aj = 0.04$ and the value of the hopping parameter $\kappa = 0.168$. The second ensemble was generated on the same system size, β and j using $N_f = 4$, resulting in a significantly finer lattice with $\sqrt{\sigma}a = 0.138(4)$ [3]. This time κ was chosen to be 0.158; both