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On the two-loop hexagon Wilson loop remainder function in $N = 4$ SYM

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ABSTRACT

A duality relation has been proposed between the planar gluon MHV amplitudes and light-like Wilson loops in $N = 4$ super Yang–Mills. At six-point two-loop, the results for the planar gluon MHV amplitude and for the light-like Wilson loop agree, but they both differ from the Bern–Dixon–Smirnov ansatz by a finite remainder function. Recently Del Duca, Duhr and Smirnov presented an analytical result for the two-loop hexagon Wilson loop remainder function in general kinematics. Their result is rather lengthy, and the dependence on the conformal cross ratios appears in a complicated way. Here we present an alternate, more compact representation for the two-loop hexagon Wilson loop remainder function.

1. Introduction

In the past few years much progress has been made in understanding scattering amplitudes in gauge theories, and in particular in $N = 4$ supersymmetric Yang–Mills (SYM) theory. An interesting feature of the $N = 4$ SYM amplitudes is the all-loop iterative structure proposed by Bern, Dixon and Smirnov (BDS) [1] for the maximally-helicity-violating (MHV) planar gluon amplitudes. Their proposal is based on the observation of an iteration relation between one- and two-loop planar four-gluon MHV amplitudes by Anastasiou, Bern, Dixon and Kosower (ABDK) [2] and an explicit computation of the four-gluon amplitude at three loops in Ref. [1].

The planar gluon amplitudes can be factorized into a universal infrared (IR) divergent factor and a finite part. Given the well-known structure of the IR divergences of gluon amplitudes [3], the BDS ansatz proposes an explicit expression for the finite part of the planar gluon MHV amplitude with an arbitrary number of external gluons, to all orders in 't Hooft coupling. An important aspect of the ansatz is that the kinematic dependence of the finite part is described by a function whose coupling dependence can be factored out, and the remaining coupling-independent part of the function is given by the finite part of the box functions entering one-loop MHV amplitude. Besides the tests for four-gluon amplitude up to three loops in Refs. [1,2], the BDS ansatz has been shown to be correct also for two-loop five-gluon amplitude [4,5].

In a remarkable paper [6], Alday and Maldacena were able to compute the planar gluon amplitudes at strong coupling using the AdS/CFT correspondence. Their result agrees with the strong coupling limit of the BDS ansatz for the four-gluon case. However, for amplitudes with a large number of external gluons, a discrepancy was found between the strong coupling prediction and the BDS ansatz [7]. This indicates a potential failure of the BDS ansatz for amplitudes with a sufficiently large number of external gluons.

Alday and Maldacena also pointed out that in the strong coupling limit the computation of planar gluon amplitudes is equivalent to the computation of the vacuum expectation value of polygonal Wilson loops with light-like edges defined by the momenta of external gluons. This suggests a duality between planar gluon amplitudes and light-like Wilson loops at strong coupling. Such a duality was then conjectured to hold at weak coupling in Ref. [8], and was verified by explicit one-loop computations for four-sided Wilson loop in the same paper and for an arbitrary n-sided case in Ref. [9]. Further two-loop results for four-, five- and six-sided Wilson loops [10–13] also found agreement with the gluon amplitude results [1,2,4,5,14]. At six-point two-loop level, both the Wilson loop and the amplitude results differ from the BDS ansatz.

The light-like n-sided Wilson loop exhibits an anomalous conformal symmetry, and the associated anomalous conformal Ward identities constrain the form of the light-like Wilson loop [11]. In general, the solution of the anomalous conformal Ward identities is uniquely determined up to a function invariant under the conformal symmetry. Such a function can be constructed from the conformally-invariant cross ratios built out of the external momenta. For $n \leqslant 5$, it is not possible to construct such conformal cross ratios due to the light-

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likeness of the external momenta. Given the duality between planar gluon amplitudes and light-like Wilson loops (which implies that the planar gluon amplitude also satisfies the anomalous conformal Ward identities), this provides an explanation for the correctness of the BDS ansatz for four- and five-gluon amplitudes: it satisfies the anomalous conformal Ward identities, and is unique because of the lack of conformal cross ratios. For $n \geqslant 6$, the conformal cross ratios can be constructed (for $n = 6$, there are 3 such ratios), therefore a solution to the anomalous conformal Ward identities can differ from the BDS form by a function of the conformal cross ratios.

Explicit numerical computations for the two-loop hexagon Wilson loop [13,15] indeed showed that the complete result differs from the BDS ansatz by a non-trivial finite remainder function, which depends only on the conformal cross ratios. In Ref. [15], also the two-loop seven- and eight-sided Wilson loops were evaluated numerically, and the corresponding remainder functions were shown to depend on the conformal cross ratios only.

In addition to the numerical computation, analytical evaluations of the remainder function have also been carried out recently, both at strong and weak couplings. In Ref. [16], the remainder function was evaluated analytically at strong coupling in a special kinematic regime where only even-sided Wilson loops are admitted and where the number of independent cross ratios is reduced so that the function is non-trivial only for $n \geqslant 8$. The explicit form of the octagon Wilson loop remainder function in this special kinematics was also given there. Later on, a numerical evaluation of the octagon Wilson loop remainder function at two loops was carried out [17] in the same kinematics and compared to the strong coupling result. The numerical comparison suggests a linear relation between the remainder functions at weak and strong couplings. In Ref. [18], Alday, Gaiotto and Maldacena computed analytically the hexagon Wilson loop remainder function in a kinematic regime where the three conformal cross ratios are equal, and found a fairly simple functional form. Recently, Del Duca, Duhr and Smirnov presented an analytical result for the two-loop hexagon Wilson loop remainder function at weak coupling [19,20], starting from the Feynman integrals contributing to the two-loop hexagon Wilson loop given in Ref. [15]. They considered the hexagon Wilson loop in the quasi-multi-Regge kinematics [21,22] where the Wilson loop exhibits exact Regge factorization,[1] therefore the analytic dependence of the remainder function on the conformal cross ratios is not modified by going to this kinematics, but the computation of the remainder function simplifies remarkably.

The two-loop hexagon Wilson loop remainder function was expressed in Refs. [19,20] in terms of transcendental weight four terms constructed from Goncharov polylogarithms and harmonic polylogarithms. The result is rather lengthy and the dependence on the conformal cross ratios appears in a complicated way.[2] In order to extract interesting physical information from the remainder function and to eventually find a systematic way to fix the BDS ansatz, a fairly simple representation of the remainder function is desirable. Moreover, the numerical results of Ref. [17] suggest a potential link between the remainder functions at weak and strong couplings. Given the simplicity of the hexagon Wilson loop remainder function at strong coupling [18], one desires to have a simple representation also for the hexagon Wilson loop remainder function at weak coupling, in order to make a comparison with the strong coupling result. In this Letter we present an alternate, more compact representation for the two-loop hexagon Wilson loop remainder function, based on the observation that the conformal-cross-ratio-dependent terms in the BDS ansatz has a simple integrand structure when written in an integral form, and also on the result of [19] as well as on the general properties of multiple polylogarithms described in Refs. [24,25].

The Letter is organized as follows. In Section 2 we briefly review the proposed duality between the planar gluon MHV amplitude and the light-like Wilson loop, and the definition of remainder function. In Section 3 we present our representation for the two-loop hexagon Wilson loop remainder function, both in the kinematic configurations where the three conformal cross ratios coincide and in general kinematics. Our conclusion is given in Section 4.

2. The planar gluon amplitude/Wilson loop duality and the remainder function

The color-ordered planar gluon MHV amplitude in $N = 4$ SYM can be factorized and written as

$$\ln \mathcal{M}_n^{(\mathrm{MHV})} = Z_n^{\mathrm{IR,div}} + F_n^{(\mathrm{MHV})}(p_1, \ldots, p_n; a) + \mathcal{O}(\epsilon), \tag{1}$$

where the left-hand side is the logarithm of the rescaled amplitude, which is defined as the ratio of the color-ordered amplitude and the corresponding tree amplitude. $Z_n^{\mathrm{IR,div}}$ represents the IR divergences of the amplitude, regularized by dimensional reduction in $D = 4 - 2\epsilon$ ($\epsilon < 0$) dimensions. $F_n^{(\mathrm{MHV})}$ is the finite part depending on the momenta of external gluons p_i ($i = 1, \ldots, n$) and on the 't Hooft coupling $a = g^2 N/(8\pi^2)$. In $N = 4$ SYM, the IR divergence is characterized by the universal cusp anomalous dimension (for the leading poles) and the collinear anomalous dimension (for the subleading poles). The finite part $F_n^{(\mathrm{MHV})}$ can be extracted from direct computation of planar n-gluon MHV amplitude, which can be carried out, e.g. with the unitarity-cut techniques [26,27]; and compared with the BDS proposed form.

On the other hand, the light-like Wilson loop dual to the planar gluon amplitude is given by

$$W(C_n) = \frac{1}{N}\langle 0|\,\mathrm{Tr}\, P \exp\left(i \oint_{C_n} dx^\mu A_\mu(x)\right)|0\rangle, \tag{2}$$

where the gauge fields $A_\mu(x)$ are integrated along the light-like polygonal contour C_n with n cusps x_i^μ, the difference of which is given by the momenta of external gluons in the dual planar gluon amplitude as

$$x_i^\mu - x_{i+1}^\mu = x_{i,i+1}^\mu = p_i^\mu. \tag{3}$$

The Wilson loop defined above has ultraviolet (UV) or cusp divergences, due to the presence of cusps on the integration contour C_n.

[1] An exact Regge factorization of four-sided Wilson loop has been observed in Ref. [8].

[2] By evaluating the integrals contributing to the two-loop hexagon Wilson loop collected in Ref. [13], one ends up with an expression [23] for the remainder function of comparable size to that in Refs. [19,20].