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First and second quantization theories of open p-brane and their spectra

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ABSTRACT

Utilizing the free Polyakov action of open p-brane, we obtain the solution of satisfying the boundary conditions to Euler–Lagrange equation of the open p-brane, the first and second quantization theories of the p-brane are given. Further, we obtain a series of new multiple commutative relations between the different normal modes of the p-brane, obtain the new lowering and raising multiple operators, and give a series of the new fundamental multiple commutative relations of the lowering and raising multiple operators in the state space. And then we mainly take $p = 3$ case for example to investigate the spectrum of the open 3-brane at different levels. Interestingly, we find three types of tachyon states including scalar states, vector states and 2-rank tensor states, respectively. Besides, the graviton fields, Kalb–Ramond fields, dilaton and photon states, 3-rank tensor states as well, appear at the same level in the open 3-brane model. For the spectrum of the p-brane ($p > 3$), one can do the whole analogous research on $p = 3$ except more complex.

1. Introduction

String theory has been probably providing the most promising descriptions of nature, especially the realizations of spectra of particles in Standard Model. Besides, some kind of particles of spin two was assigned to gravitons [1–4]. On the analogy with particles, the dynamical properties of strings are described by Nambu–Goto action [5], given in terms of the area of string worldsheet. By involving an auxiliary worldsheet metric, it can also be replaced by a classically equivalent action, called Polyakov action with local conformal symmetries [6]. In contrast with the non-polynomial Nambu–Goto action the new action is quadratic in the derivatives of the coordinates. Some authors have investigated the connections between the two kinds of actions by introducing the interpolating actions [7–9].

Starting with the Letter of Kikkawa and Yamasaki [10], we have had an increasing interest in the theory of (two and more)-dimensional extended objects (membranes and p-brane) as unified theories containing non-abelian excitations [11]. The high-dimensional objects that motivated recent progress in string theory are extended structures embedded in a higher-dimensional spacetime from which it inherits an induced metric [12–15]. Over the last decade string theory has been gradually replaced by M-theory as the natural candidate for a fundamental description of nature. While a complete definition of M-theory is yet to be given, it is believed that the five perturbatively consistent string theories are different phases of this theory. Indeed it is known that membrane and five-brane occur naturally in eleven-dimensional supergravity, which is argued to be the low-energy limit of M-theory. Also, string theory is effectively described by the low-energy dynamics of a system of branes. For instance, the membrane of M-theory may be "wrapped" around the compact direction of radius R to become the fundamental string of type-IIA string theory, in the limit of vanishing radius [16–22]. A fundamental type-IIB string can be thought of as an M2-brane wrapped around x^{10}.

Ref. [22] gives the scheme of Dirac quantization of open p-brane in the D-brane background. In Ref. [23], we had discussed the quantization and spectrum of open 2-brane, from which we had seen more contents appeared than string case, especially the appearance of two types of tachyon states. In this Letter, we generalize the results to higher extended objects such as 3-brane and p-brane. As before, we also mainly work in 26-dimensional Minkowski spacetime, the arrangement is: Section 2 is the solution to Euler–Lagrange equation of p-brane and its quantization; Section 3 is the discussions on the second quantization of the

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p-brane; in Section 4, we give Hamiltonian of the open 3-brane and deduce its representation in terms of normal modes; in Section 5, we define the vacuum state and get a series of excited states by acting the raising operators on it and discuss some lower levels. Section 6 is summary and conclusion.

2. The solution to Euler–Lagrange equation of p-brane and its quantization

An open p-brane is a p-dimensional object which sweeps out a $(p+1)$-dimensional world volume parameterized by $\tau, \sigma^1, \ldots, \sigma^p$. And these parameters are collectively referred as ξ_i $(i = 0, 1, 2, \ldots, p)$. Then Polyakov action for the p-brane is given by [7]

$$S_P = -\frac{1}{4\pi\alpha'}\int d^{p+1}\xi\,\sqrt{-h}\big[h^{ab}\partial_a X^\mu \partial_b X_\mu - (p-1)\big], \tag{1}$$

where $h = \det|h_{ab}|$. It is well known that Eq. (1) is equivalent to Nambu–Goto action of the p-brane, and there are other lagrangian formalisms of p-branes, which do not require the fine tunned cosmological term [24]. Because $h = h_{ab}\Delta_{ab}$, $h^{ab} = \Delta_{ba}/h$, we have [1,2]

$$\delta h = -hh_{ab}\delta h^{ab}, \tag{2}$$

$$\delta\sqrt{-h} = -\frac{1}{2}\sqrt{-h}h_{ab}\delta h^{ab}, \tag{3}$$

$$\delta\big(\sqrt{-h}h^{cd}\big) = \sqrt{-h}\left(\delta h^{cd} - \frac{1}{2}h_{ab}h^{cd}\delta h^{ab}\right). \tag{4}$$

Let $\gamma_{ab} = \partial_a X^\mu \partial_b X_\mu$, Euler–Lagrange equation for h^{ab}

$$\frac{\delta S}{\delta h^{ab}} - \partial_c\frac{\delta S}{\delta(\partial_c h^{ab})} = 0 \tag{5}$$

gives

$$\frac{\delta S}{\delta h^{ab}} = -\frac{\sqrt{-h}}{2\pi\alpha'}\left\{\gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{p-1}{2}h_{ab}\right\} = 0. \tag{6}$$

Define the energy–momentum tensor as

$$T_{ab} = \frac{-2\pi\alpha'}{\sqrt{-h}}\frac{\delta S}{\delta h^{ab}} = \gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{p-1}{2}h_{ab}. \tag{7}$$

Because quantization of gravity field cannot very well be finished up to now, without losing the character of the open p-brane second quantization, we can choose the metric h_{ab} as $(-, +, +, \ldots, +)$, then we have

$$T_{00} = \frac{1}{2}(\gamma_{00} + \gamma_{11} + \gamma_{22} + \cdots + \gamma_{pp} - p + 1) = 0, \tag{8}$$

$$T_{11} = \frac{1}{2}(\gamma_{00} + \gamma_{11} - \gamma_{22} - \cdots - \gamma_{pp} + p - 1) = 0, \tag{9}$$

$$T_{22} = \frac{1}{2}(\gamma_{00} - \gamma_{11} + \gamma_{22} - \cdots - \gamma_{pp} + p - 1) = 0, \tag{10}$$

$$\vdots$$

$$T_{pp} = \frac{1}{2}(\gamma_{00} - \gamma_{11} - \gamma_{22} - \cdots + \gamma_{pp} + p - 1) = 0. \tag{11}$$

Eq. (8) indicates that the Hamiltonian of p-brane system vanishes, which we will discuss later. Eqs. (9)–(11) may be viewed constraint equations.

The Euler–Lagrange equation for X_μ can be derived from the variational principle as follows

$$\left(\partial_\tau^2 - \sum_{i=1}^{p}\partial_i^2\right)X^\mu(\tau, \sigma^1, \sigma^2, \ldots, \sigma^p) = 0 \tag{12}$$

with the Neumann boundary conditions

$$\partial_i X^\mu(\tau, \sigma^1, \ldots, \sigma^p)\big|_{\sigma^i=0} = \partial_i X^\mu(\tau, \sigma^1, \ldots, \sigma^p)\big|_{\sigma^i=\pi} = 0 \quad (i = 1, 2, \ldots, p). \tag{13}$$

On the other hand, because general physical processes should satisfy quantitative causal relation [25,26], some changes (cause) of some quantities in Eq. (12) must lead to the relative some changes (result) of the other quantities in Eq. (12) so that Eq. (12)'s right side keeps no-loss-no-gain, i.e., zero, namely, Eq. (12) also satisfies the quantitative causal relation, which just makes X^I relative to $(\tau, \sigma^1, \sigma^2, \ldots, \sigma^p)$ to form a coupling physical system of different variables.

The canonical momenta for canonical variables X_μ are defined as

$$P^\mu = \frac{\partial\mathcal{L}}{\partial\dot{X}_\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu. \tag{14}$$

Therefore, we find out the solution satisfying the boundary conditions to the Euler–Lagrange equation as follows

$$X^0 = \frac{x^0}{\pi^{\frac{(p-1)}{2}}} + \frac{2\alpha' p^0}{\pi^{\frac{(p-1)}{2}}}\tau, \qquad X^1 = \frac{x^1}{\pi^{\frac{(p-1)}{2}}} + \frac{2\alpha' p^1}{\pi^{\frac{(p-1)}{2}}}\tau, \tag{15}$$

$$\begin{aligned} X^I(\tau, \sigma^1, \ldots, \sigma^p) &= \frac{x^I + 2\alpha' p^I\tau}{\pi^{\frac{(p-1)}{2}}} + i\sqrt{2\alpha'}\sum_{n_1,\ldots,n_p=0}^{+\infty}\left(\sum_{i=1}^{p}n_i^2\right)^{\frac{-1}{4}} \\ &\quad\times\left(X^I_{n_1n_2\cdots n_p}e^{i\tau\sqrt{\sum_{i=1}^{p}n_i^2}} - \big(X^I_{n_1n_2\cdots n_p}\big)^\dagger e^{-i\tau\sqrt{\sum_{i=1}^{p}n_i^2}}\right) \\ &\quad\times\prod_{i=1}^{p}\cos n_i\sigma^i, \end{aligned} \tag{16}$$

$$\begin{aligned} P^J(\tau, \sigma^1, \ldots, \sigma^p) &= \frac{1}{\pi}\Bigg[\frac{p^J}{\pi^{\frac{(p-1)}{2}}} + \sqrt{\frac{2}{\alpha'}}\sum_{n_1,n_2,\ldots,n_p=0}^{+\infty}\left(\sum_{i=1}^{p}n_i^2\right)^{\frac{1}{4}} \\ &\quad\times\left(\big(P^J_{n_1n_2\cdots n_p}\big)^\dagger e^{i\tau\sqrt{\sum_{i=1}^{p}n_i^2}} + P^J_{n_1n_2\cdots n_p}e^{-i\tau\sqrt{\sum_{i=1}^{p}n_i^2}}\right) \\ &\quad\times\prod_{i=1}^{p}\cos n_i\sigma^i\Bigg], \end{aligned} \tag{17}$$

where we have fixed the first two directions of the solution by two constraints of Eqs. (9)–(11), i.e. Eq. (15), which may make us able to gain reasonable spectrum of lower energy levels. And we have introduced $(X^I_{n_1n_2\cdots n_p})^\dagger$ and $(P^J_{n_1n_2\cdots n_p})^\dagger$ as Hermitian operators for $X^I_{n_1n_2\cdots n_p}$ and $P^J_{n_1n_2\cdots n_p}$, respectively, in order to guarantee the Hermiticity of $X^I(\tau, \sigma^1, \ldots, \sigma^p)$ and $P^I(\tau, \sigma^1, \ldots, \sigma^p)$, and $\{n_i\}$ cannot be zero simultaneously. Now there are still $p-2$ constraints left, which will restrict the p-brane on the $(D-p+2)$-dimensional hypersurface in D-dimensional Minkowski spacetime for $p \geqslant 2$. In this sense, we can conclude that $D - p + 2 \geqslant p + 1$, i.e., $p \leqslant \frac{D+1}{2}$.

Using (14), (16) and (17), we set up the following relations

$$X^I_{n_1n_2\cdots n_p} = -2\big(P^I_{n_1n_2\cdots n_p}\big)^\dagger; \qquad \big(X^I_{n_1n_2\cdots n_p}\big)^\dagger = -2P^I_{n_1n_2\cdots n_p}. \tag{18}$$

In order to determinate the commutative relations, we must calculate the commutative relations of $X^I(\tau, \sigma^1, \ldots, \sigma^p)$ and $P^J(\tau, \sigma^1, \ldots, \sigma^p)$ basing on the Delta function as [3]

$$\delta(\sigma - \sigma') = \frac{1}{\pi}\left(1 + 2\sum_{n=1}^{+\infty}\cos n\sigma\cos n\sigma'\right), \tag{19}$$