Physics Letters B 699 (2011) 271–275

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The Sommerfeld enhancement for scalar particles and application to sfermion co-annihilation regions

Andrzej Hryczuk

SISSA and INFN – Sezione di Trieste, via Bonomea 265, 34136 Trieste, Italy

ARTICLE INFO

Article history:
Received 24 February 2011
Received in revised form 7 April 2011
Accepted 8 April 2011
Available online 14 April 2011
Editor: A. Ringwald

Keywords:
Dark matter
Non-perturbative effects

ABSTRACT

We study the impact of the Sommerfeld enhancement on the thermal relic density of the lightest neutralino in the case of large co-annihilation effects with a scalar particle. The proper way of including the Sommerfeld effect in this case is discussed, and the appropriate formulas for a general scenario with a set of particles with arbitrary masses and (off-)diagonal interactions are provided. We implement these results to compute the relic density in the neutralino sfermion co-annihilation regions in the mSUGRA framework. We find non-negligible effects in whole sfermion co-annihilation regimes. For stau co-annihilations the correction to the relic density is of the order of several per cent, while for stop co-annihilations is much larger, reaching a factor of 5 in some regions of the parameter space. A numerical package for computing the neutralino relic density including the Sommerfeld effect in a general MSSM setup is made public available.

1. Introduction

In recent years, the existence of dark matter (DM) has been well established and its density measured with a few per cent level of precision. A recent analysis, within the 6-parameter ΛCDM model, of the 7-year WMAP data [1], combined with the Baryon Acoustic Oscillations [2] and the recent redetermination of the Hubble constant [3], gives [4]: $\Omega_{\mathrm{CDM}}h^2 = 0.1123 \pm 0.0035$, where Ω is the ratio between mean density and critical density, and h is the Hubble constant in units of 100 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

One of the most attractive scenarios explaining the nature of dark matter is that it is composed of stable weakly interacting massive particles (WIMPs), since their thermal relic abundance is naturally of the order of the measured one. However, in order to predict its precise value for a given model a careful calculation has to be done. In particular, there are several effects that can alter the relic density of a thermal relic and which need to be taken into account.

In this Letter we contribute to the study of one of these effects, i.e. the Sommerfeld enhancement [5]. It is a non-relativistic effect changing the annihilation cross section due to a long range force acting between slowly moving initial particles. This effect has been studied widely recently (see e.g. [6] and references therein). Although its applications were discussed for both scalar and fermion initial states, the derivation of Sommerfeld corrections for a general multi-state case was given explicitly only for fermions [7] (see also [8] for a different approaches). Here we extend it to the case of scalar–scalar and fermion–scalar pairs and discuss possible applications. Finally, we present some numerical results for the impact of the Sommerfeld effect on the relic density of the neutralino in stau and stop co-annihilation regions in minimal supergravity (mSUGRA) scenario.

2. Sommerfeld enhancement for scalars

We consider an annihilation process of two particles, φ_i and φ_j, which are coupled to some light interaction boson ϕ, leading to a long range interaction between them. In the case when this interaction is diagonal (i.e. the exchange of ϕ does not change the particles), in the non-relativistic limit the spin of initial particles does not matter – the static force is the same for both scalars and fermions. This is however not true if interactions can be off-diagonal and intermediate particles can have different masses. Then due to the differences in the couplings and propagators between scalars and fermions, the computations of the Sommerfeld effect slightly differ.

In deriving the effect for this case we follow the approach of Ref. [7], where a general method of computing the Sommerfeld enhancement from the field theory diagrams was presented: to obtain the Sommerfeld enhancement factors S_{ij} one has to solve the set of Schrödinger equations for the two-body wave-functions ψ_{ij}:

$$-\frac{\partial^2}{2m_r^{ij}}\psi_{ij}(\vec{r}) = U^0_{ij}(\vec{r}) + (\mathcal{E} - 2\delta m_{ij})\psi_{ij}(\vec{r}) + \sum_{i'j'\phi} V^{\phi}_{ij,i'j'}\psi_{i'j'}(\vec{r}), \tag{1}$$

E-mail address: hryczuk@sissa.it.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.04.016

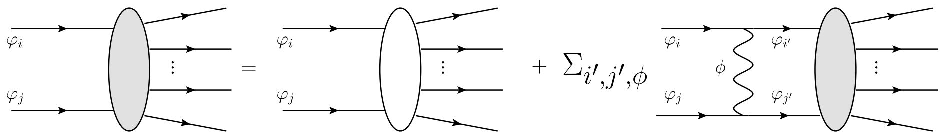

Fig. 1. Recursive relation for the full annihilation amplitude including the Sommerfeld enhancement. Blob represents any possible annihilation process.

and then compute

$$S_{ij} = \left| \frac{\psi_{ij}(\infty)}{\psi_{ij}(0)} \right|^2, \tag{2}$$

where m_r^{ij} is the reduced mass of annihilating $\varphi_i\varphi_j$ pair, U_{ij}^0 contains the tree-level amplitude and the sum is over (possibly different) $\varphi_{i'}\varphi_{j'}$ intermediate states and different interactions. Here $\mathcal{E} = \vec{p}^{\,2}/2m_r^{ab}$ is the kinetic energy of the incoming pair (at infinity), with m_r^{ab} its reduced mass and $\vec{p}$ the CM three-momentum; $2\delta m_{ij} = m_i + m_j - (m_a + m_b)$ is the mass splitting (for more details see Ref. [6]). The potential has the form:

$$V^{\phi}_{ij,i'j'}(r) = \frac{c_{ij,i'j'}(\phi)}{4\pi} \frac{e^{-m_\phi r}}{r}, \tag{3}$$

where $c_{ij,i'j'}(\phi)$ are coefficients depending on the couplings and states involved. For incoming fermions the coefficients were presented in Table 1 of Ref. [6]. Below we give results for scalar–scalar and fermion–scalar pairs.

To obtain those coefficients we use a method which is a straightforward generalization of the one developed in Ref. [7]. We write a recurrence relation for the annihilation amplitudes as visualized in Fig. 1. Assuming that $\delta m_{ij}/m_i \ll 1$, in the non-relativistic limit we can transform this integral equation to the Schrödinger one, obtaining automatically the coefficients $c_{ij,i'j'}(\phi)$ in the potential.

Let's denote by a superscript S the case with two scalars and by F with one scalar and one fermion. In the second case let i and i' be fermions and j and j' scalars. Then we find:

$$c_{ij,i'j'} = g^{\phi}_{ii'} g^{\phi}_{jj'} N^{S,F}_{ij,i'j'} A^{S,F}_{\phi}(m_i, m_j, m_{i'}, m_{j'}), \tag{4}$$

where $g^{\phi}_{ii'}$ is a coupling present in the $ii'\phi$ vertex; the normalization and combinatorics gives

$$N^S_{ij,i'j'} = \begin{cases} 1, & i = j, i' = j' \text{ or } i \neq j, i' \neq j', \\ \sqrt{2}, & i \neq j, i' = j' \text{ or } i = j, i' \neq j', \end{cases}$$

$$N^F_{ij,i'j'} = 1,$$

and factors $A^{S,F}_{\phi}$ are, with $\phi = V, A, S$ indicating respectively a vector, an axial vector and a scalar:

$$A^S_V = A^S_A = \frac{1}{2}\left(1 + \frac{m_i}{2m_{i'}} + \frac{m_j}{2m_{j'}}\right), \tag{5}$$

$$A^S_S = \frac{1}{4m_{i'}m_{j'}}, \tag{6}$$

$$A^F_V = \frac{m_{j'} + m_j}{2m_{j'}}, \tag{7}$$

$$A^F_A = 0, \tag{8}$$

$$A^F_S = \frac{1}{2m_{j'}}. \tag{9}$$

In the limit when all the masses are equal coefficients A^S_V, A^S_A reduce to the ones which were used in Refs. [9–11]. However, in general case when the masses of intermediate scalars differ, Eqs. (5) and (6) have to be used. Neglecting this fact can give rise to several percent difference in the $c_{ij,i'j'}(\phi)$ coefficients.[1] We stress once again that those results are valid in the non-relativistic limit and when mass splitting is much smaller than all of the masses involved.

All the considerations above implicitly assumed that the interaction strength is sufficiently weak that the higher loop corrections do not alter the potential significantly. This is true for the weak and electromagnetic interactions, as well as for the Higgs exchange. However, in the case of strong interactions, corrections to the gluon exchange coming from gluon self-interactions and fermion loops may become important. To take this into account, following [9], instead of the potential (3) we will use one computed in [12], which in the configuration space is[2]:

$$V(\vec{r}) = -C_F \frac{\alpha_s}{r} - C_F \frac{\alpha_s^2}{4\pi} \frac{1}{r}\left[\frac{31}{9} C_A - \frac{20}{9} T_F n_f + \beta_0\left(2\gamma_E + \log\left(\mu^2 r^2\right)\right)\right] + \mathcal{O}\left(\alpha_s^3\right), \tag{10}$$

where $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$, n_f is the number of massless quarks (we choose it to be 5, since the stop co-annihilation is most important in the $\mathcal{O}(100\ \text{GeV})$ region, where the top mass m_t is non negligible) and Euler gamma is $\gamma_E \approx 0.5772$. For the case of SU(3) we have $C_F = 4/3$, $T_F = 1/2$, $C_A = 3$. For the QCD scale we take $\mu^2 = 2m_t^2$.

Another effect one has to take into account is the presence of thermal corrections, as first discussed in a similar context in [10]. They change the exchanged boson masses and in particular photon and gluon become massive, which introduces a Yukawa cut-off to the potential in Eq. (10). These corrections may be also important for the mass splittings, if there are nearly degenerate states present in the spectrum.

There are two types of thermal effects which we need to include as discussed in [10,6]: the scaling of the Higgs VEV with the temperature [13], $v(T) = v\,\text{Re}\sqrt{1 - T^2/T_c^2}$ where we took $T_c = 200$ GeV, and the contribution to gauge boson masses due to the screening by the thermal plasma; the so-called Debye mass [14]. For the gluon the screening of the plasma introduces at a leading order a contribution [15]:

$$m_g^2 = (N_c/3 + N_f/6) g_s^2 T^2 = \frac{3}{2} g_s^2 T^2. \tag{11}$$

3. Applications

To have an idea when the Sommerfeld effect can have a non-negligible impact on the relic density it is useful to give some approximate general conditions, which have to be satisfied by the dark matter particle or the co-annihilating one: i) coupling to the boson with much lower mass ("long range force"), ii) the coupling strength at least of the order of the weak coupling and iii) if the

[1] Some of these coefficients are divergent in the limit when one of the masses vanish. However, in this case the non-relativistic approximation does not hold and hence these results are not valid.

[2] Note that here the interaction is diagonal and one does not need to use coefficients (5)–(9).