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4D gravity on a BPS brane in 5D *AdS*-Minkowski space

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ABSTRACT

We calculate small correction terms to gravitational potential near an asymmetric BPS brane embedded in a 5D *AdS*-Minkowski space in the context of supergravity. The normalizable wave functions of gravity fluctuations around the brane describe only massive modes. We compute such wave functions analytically in the thin wall limit. We estimate the correction to gravitational potential for small and long distances, and show that there is an intermediate range of distances in which we can identify 4D gravity on the brane below a crossover scale. The 4D gravity is metastable and for distances much larger than the crossover scale the 5D gravity is recovered.

Following the original idea of Randall and Sundrum (RS) [1], we consider a braneworld scenario. In the RS scenario the five-dimensional gravity is coupled to a negative cosmological constant and a 3-brane sourced by a delta function. The solution in such setup is a *symmetric* solution given in terms of two copies of AdS_5 spaces patched together along the 3-brane. Although in this setup the fifth dimension is infinite the volume of the 5D bulk space is finite because the geometry is warped. As a consequence this allows having graviton zero mode responsible for 4D gravity on the brane. This is not necessary true for spaces asymptotically flat, because no zero mode emerges anymore. This was first shown by Gregory–Rubakov–Sibiryakov (GRS) [2] and Dvali–Gabadadze–Porrati (DGP) [3]. The nice consequence of such an alternative setup is that 4D gravity on the 3-brane now emerges due to gravity massive modes and then is metastable. However, gravity massive modes can live long enough before escaping from the 3-brane to produce 4D gravity within a sufficient large scale — the crossover scale.

In the present study we investigate such a scenario in a consistently truncated 5D supergravity [4,5], where the 3-brane appears as BPS solutions. They are solutions of first-order equations that emerge through Killing spinor equations that preserve part of the supersymmetries and also satisfy Einstein equations. We shall focus on the bosonic sector with 5D gravity coupled to two real scalar fields [6,7].

In our investigations we are mainly interested on induced 4D gravity on *asymmetric* brane solutions [8–11]. Such brane solutions have naturally appeared in the supergravity context in four [8] and five dimensions [7] where the thick 3-brane is embedded in an asymptotically five-dimensional *AdS*–Minkowski space. We shall consider the later case, because it allows the possibility of metastable 4D gravity as first pointed out in the GRS [2] and DGP [3] scenarios. Because the five-dimensional space is asymptotically Minkowski on one side of the 3-brane its volume is infinite and then no gravity zero mode emerges. However, just as in GRS and DGP scenarios, we also have found 4D gravity that lives long enough within the crossover scale.

Thus, as emphasized in GRS and DGP scenarios, we shall focus on the main beautiful characteristic of the 4D metastable gravity, that is the fact that whereas gravity becomes four-dimensional for distances very much smaller than the crossover scale, it emerges as a five-dimensional gravity for distances very much larger than such scale. In doing so, we shall find the Newtonian potential induced by the gravity massive modes of a Schroedinger-like equation for the gravity fluctuations around the asymmetric 3-brane solution.

Le us consider the bosonic sector of the supergravity action for spacetimes in arbitrary D dimensions ($D > 3$) coupled to $\mathcal{N}$ real scalar fields given by [4,5]

$$S = \int d^D x \sqrt{|g|} \left[\frac{1}{2\kappa^{D-2}} R - \frac{1}{2} g^{MN} \partial_M \phi_i \partial_N \phi_i - V(\phi_i) \right], \qquad (1)$$

where $\kappa = \frac{1}{M_*}$ is the D-dimensional Planck length, and the potential of the scalar fields is taken as

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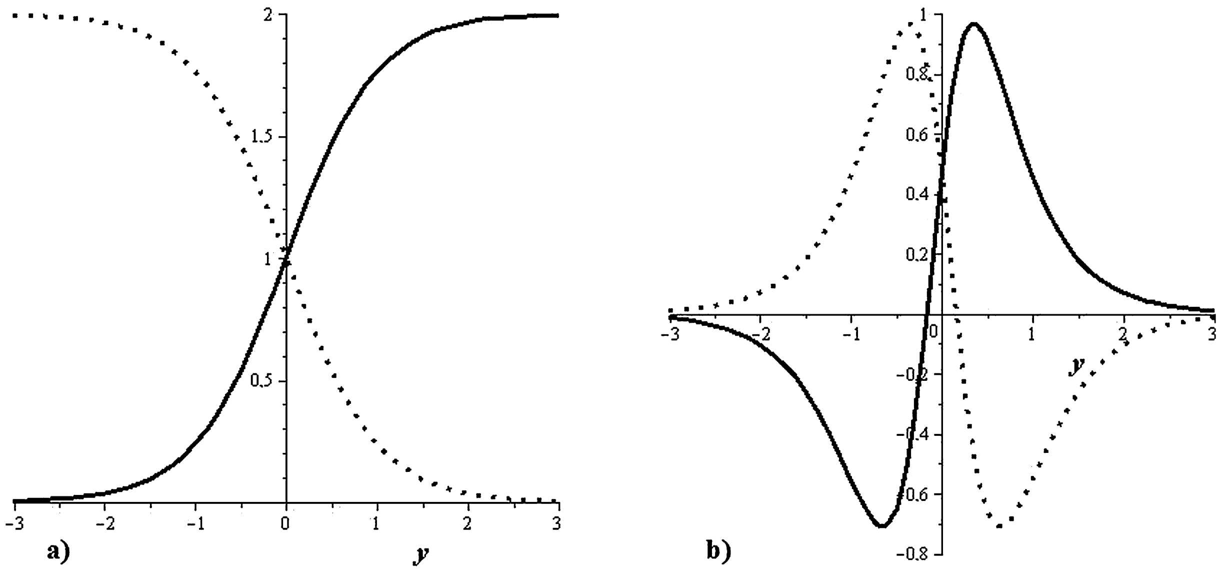

Fig. 1. a) Warp factor for BPS asymmetric branes connecting the spaces AdS_5–$\mathbb{M}_5$ (solid line) and $\mathbb{M}_5$–AdS_5 (dashed line) asymptotically, with $a = 1$ and $b = 2/\sqrt{3}$; b) corresponding energy densities.

$$V(\phi_i) = 2(D-2)^2\left[\left(\frac{\partial W}{\partial \phi_i}\right)^2 - \kappa^{D-2}\left(\frac{D-1}{D-2}\right)W^2\right], \qquad (2)$$

where $W(\phi_i)$ is the superpotential, and ϕ_i, $i = 1, 2, \ldots, \mathcal{N}$, are the scalar fields. We employ the generalized Randall–Sundrum metric:

$$ds^2_D = e^{A(y)}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dy^2, \qquad (3)$$

where $e^{A(y)}$ is warp factor, $\mu, \nu = 0, 1, 2, \ldots, D-2$ are indices on the $(D-2)$-brane. By using the action (1) with the metric (3), we obtain the set of equations

$$\phi_i'' + (D-1)A'\phi_i' - \frac{\partial V}{\partial \phi_i} = 0, \qquad (4a)$$

$$A'' = -\frac{\kappa^{D-2}}{(D-2)}\phi'^2 \qquad (4b)$$

and

$$A'^2 = 2\frac{\kappa^{D-2}}{(D-1)(D-2)}\left(\frac{1}{2}\phi'^2 - V\right), \qquad (4c)$$

which are solved by the following first order equations obtained from the Killing spinor equations

$$\partial_y A = \mp 2\kappa^{D-2}W, \qquad (5a)$$

$$\partial_y \phi_i = 2(D-2)\frac{\partial W}{\partial \phi_i}, \qquad (5b)$$

assuming that the scalar fields only depend on the transverse coordinate y. The graviton modes on $(D-2)$-branes are governed by a linearized gravity equation of motion in arbitrary number of dimensions $(D > 3)$ given by [4,5]

$$\partial_M\left(\sqrt{-g}g^{MN}\partial_N \Phi\right) = 0, \qquad (6)$$

where Φ describes the wave function of the graviton on non-compact coordinates $M, N = 0, 1, 2, \ldots, D-1$. Let us consider $\Phi = h(y)\varphi(x^\mu)$ into (6) and the fact that $\Box_{D-1}\varphi = m^2\varphi$, where $\Box_{D-1}$ is the flat Laplacian operator on the tangent frame. Thus, the wave equation for the graviton through the transverse coordinate y reads

$$\frac{\partial_y(\sqrt{-g}g^{yy}\partial_y h(y))}{\sqrt{-g}} = -m^2\left|g^{00}\right|h(y). \qquad (7)$$

This is our starting point to investigate both zero and massive graviton modes on the branes. Using the components of the metric (3) into Eq. (7) we have

$$\frac{1}{2}(D-1)\partial_y A\partial_y h(y) + \partial_y h(y) = -m^2 e^{-A(y)}h(y), \qquad (8)$$

which, changing the metric (3) by the conformally flat metric,

$$ds^2_D = e^{A(z)}\left(\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dz^2\right), \qquad (9)$$

and employing the changes of variables: $h(y) = \psi(z)\,e^{-\frac{A(z)(D-2)}{4}}$ and $z(y) = \int e^{-\frac{A(y)}{2}}\,dy$, can be written as the Schroedinger-like equation,

$$-\partial_z^2\psi(z) + V(z)\psi(z) = m^2\psi(z), \qquad (10)$$

where the potential $V(z)$ is given by

$$V(z) = \frac{(D-2)^2}{16}(\partial_z A)^2 + \frac{D-2}{4}\partial_z^2 A. \qquad (11)$$

Now, let us examine the theory introduced by the action (1), for five-dimensional gravity coupled to two real scalar fields, ϕ_1 and ϕ_2. So let us make the transformation $W \to W/[(D-1)(D-2)]$, with $D = 5$, in (5a) and (5b), and use units which $\kappa^{D-2} = 2$. This is to be in accord with the model presented in Ref. [7] through the superpotential $W = 3/2a\sin(\sqrt{2}b\phi_1)\cos(\sqrt{2}b\phi_2)$, where a, b are real constants — see also [13] for other superpotentials. The orbits $\cos(\sqrt{2}b\phi_1) = C\sin(\sqrt{2}b\phi_2)$, where C is a real constant, decouple the first-order equations, and for $C = 1$ give the solutions

$$\phi_1(y) = \pm\frac{\sqrt{2}}{4b}\arccos\left(\tanh\left(\frac{3}{4}ab^2 y\right)\right) + (n+1)\frac{\sqrt{2}\pi}{4b},$$

$$\phi_2(y) = \pm\frac{\sqrt{2}}{4b}\arccos\left(\tanh\left(\frac{3}{2}ab^2 y\right)\right) + n\frac{\sqrt{2}\pi}{4b}, \qquad (12)$$

such that we obtain

$$A(y) = (-1)^{n+1}\frac{ay}{2} + \frac{2}{3b^2}\ln\left(q\,\text{sech}\left(\frac{3}{4}ab^2 y\right)\right), \qquad (13)$$