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Pseudo-Newtonian potentials and the radiation emitted by a source swirling around a stellar object

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ABSTRACT

We use pseudo-Newtonian potentials to compute the scalar radiation emitted by a source orbiting a stellar object. We compare the results obtained in this approach with the ones obtained via quantum field theory in Schwarzschild spacetime. We find that, up to the marginally stable circular orbit, the potential that better reproduces the Schwarzschild results is the Nowak–Wagoner one. For unstable circular orbits, none of the pseudo-Newtonian potentials considered in our analysis produces satisfactory results. We show that the Paczyński–Wiita potential, the most used in the literature to analyze accretion disks, generates the least satisfactory results for the scalar radiation emitted by the source in circular orbit around a Schwarzschild black hole.

1. Introduction

In the late 1970s B. Paczyński and P.J. Wiita [1] proposed a way to reproduce, without the use of general relativity, some relativistic features of the Schwarzschild spacetime. This kind of approach is carried out assuming that the gravitational interaction between bodies occurs by means of a force in flat spacetime, described by potentials that became known as *pseudo-Newtonian*. These potentials, which are essentially modifications of the Newtonian potential, allowed the analysis of accretion disks around Schwarzschild black holes in the context of Newtonian gravity. Subsequent extensions of this approach have been related to several physical situations, as runaway instability [2], spacetimes with rotation [3], gravitational wave emission [4], spacetimes with cosmological constant [5], spherical accretion onto compact objects [6], acoustic perturbations on steady spherical accretion [7], coalescence of black hole-neutron star binary systems [8], chaotic phenomena [9], and others.

Among the pseudo-Newtonian potentials proposed along the last 30 years to reproduce the structure of circular orbits in black hole spacetimes, the Paczyński–Wiita one still stands out for its simplicity and accuracy. This potential can reproduce exactly the marginally stable circular orbit ($r_{ms} \equiv 6M$), as well as the marginally bounded circular orbit ($r_{mb} \equiv 4M$), for a particle rotating around a Schwarzschild black hole. Even if incoherent radiation generated by interactions within the disk may play a very important role in the energy radiated from accretion disks, it is also interesting to study the radiation emission processes related to particles in circular motion around he black hole.

In this Letter, we present a different test to the pseudo-Newtonian potentials, with an original and independent way to verify their applicability, using them in the context of radiation emission processes, in contrast to the usual studies based in classical mechanics and fluid dynamics.

We use the pseudo-Newtonian potentials proposed by Paczyński and Wiita, Nowak and Wagoner [10], and Artemova et al. [11], as well as the Newtonian potential, to compute the synchrotron scalar radiation emitted by a source orbiting a stellar object in Minkowski spacetime. We compare these results with the power emitted by a source swirling around a Schwarzschild black hole, obtained in the framework of quantum field theory in curved spacetimes. We obtain that, among the pseudo-Newtonian potentials considered in this study, the Paczyński–Wiita one gives the least satisfactory results for this radiation emission problem.

We assume natural units $c = \hbar = G = 1$ and signature $(+, -, -, -)$ throughout this Letter.

2. Pseudo-Newtonian potentials

Pseudo-Newtonian potentials are modifications of the Newtonian potential

$$\varphi_1(r) = -\frac{M}{r} \qquad (1)$$

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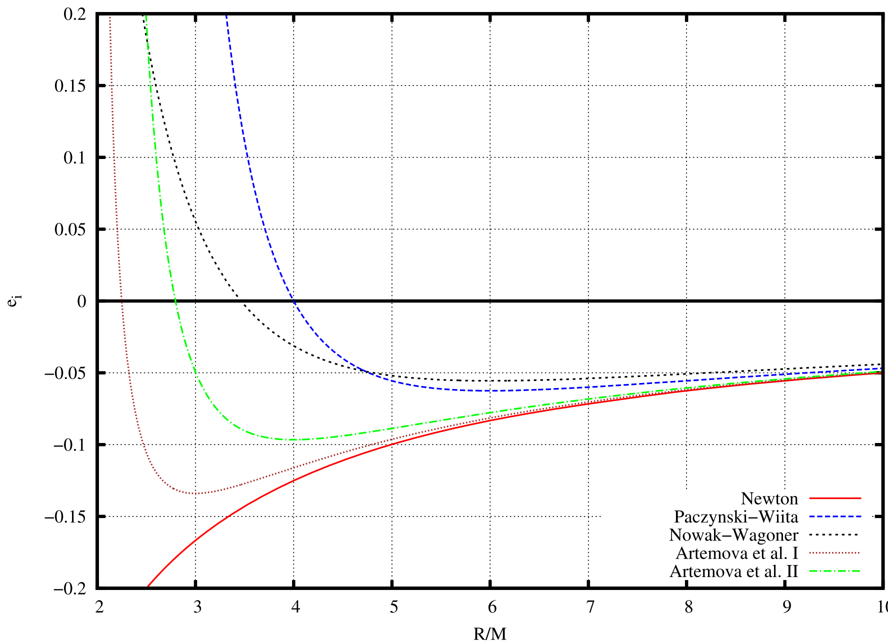

Fig. 1. The specific mechanical energy, as a function of R, for a particle in uniform circular motion around a stellar object, computed using the potentials (1)–(5).

(where M is mass of the central object), done to reproduce some relativistic aspects of particles orbiting central objects.

Here we will be concerned with the following pseudo-Newtonian potentials:

(i) Paczyński–Wiita potential [1]

$$\varphi_2(r) = -\frac{M}{r-2M}, \tag{2}$$

(ii) Nowak–Wagoner potential [10]

$$\varphi_3(r) = -\frac{M}{r}\left[1 - 3\frac{M}{r} + 12\frac{M^2}{r^2}\right], \tag{3}$$

(iii) Artemova et al. potentials [11]

$$\varphi_4(r) = -1 + \left(1 - \frac{2M}{r}\right)^{1/2}, \tag{4}$$

which we will refer to as Artemova et al. I, and

$$\varphi_5(r) = \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right), \tag{5}$$

which will be referred to as Artemova et al. II.

From these potentials, we can find the orbital structure of a particle in uniform circular motion around a stellar object in Minkowski spacetime for each case. With this aim, we use the definition of the specific mechanical energy [1]

$$e_i(r) \equiv \frac{1}{2}\dot{r}^2 + \frac{1}{2}\frac{l_i^2}{r^2} + \varphi_i(r),$$

where $i = 1,2,3,4,5$ stands for each of the potentials given by Eqs. (1), (2), (3), (4), and (5), respectively. The specific angular momentum, l_i, is given by

$$l_i(r) = \left(r^3\frac{d\varphi_i}{dr}\right)^{1/2}. \tag{6}$$

Therefore, the specific mechanical energy for each potential (1)–(5) can be written as

$$e_1(R) = -\frac{M}{2R},$$

$$e_2(R) = \left(-\frac{M}{2R}\right)\left[\frac{(R-4M)R}{(R-2M)^2}\right],$$

$$e_3(R) = \left(-\frac{M}{2R}\right)\left(1 - \frac{12M^2}{R^2}\right),$$

$$e_4(R) = -1 + \frac{M}{2R}\left(1 - \frac{2M}{R}\right)^{-1/2} + \left(1 - \frac{2M}{R}\right)^{1/2},$$

and

$$e_5(R) = \frac{M}{2R}\left(1 - \frac{2M}{R}\right)^{-1} + \frac{1}{2}\ln\left(1 - \frac{2M}{R}\right),$$

respectively. The circular bounded orbits are obtained by imposing $e_i(R) < 0$ [1], where R is the radius of the circular orbit. The marginally bounded orbit is defined as the orbit for which the mechanical energy vanishes.

In Fig. 1 we plot the specific mechanical energy for the potentials (1)–(5). We see that only the Paczyński–Wiita potential reproduces the exact value of the marginally bounded circular orbit provided by general relativity ($r_{mb} = 4M$) [12]. We also see that all orbits are bounded in the Newtonian case, as expected.

The stable orbits are such that $\frac{dl_i}{dR} > 0$ [13,14]. Using Eqs. (1)–(6), we find

$$\frac{dl_1}{dR} = \frac{1}{2}\left(\frac{M}{R}\right)^{1/2},$$

$$\frac{dl_2}{dR} = \frac{1}{2}\left(\frac{M}{R}\right)^{1/2}\left[\frac{(R-6M)R}{(R-2M)^2}\right],$$

$$\frac{dl_3}{dR} = \frac{1}{2}\left(\frac{M}{R}\right)^{1/2}\left(1 - \frac{36M^2}{R^2}\right)\left(1 - \frac{6M}{R} + \frac{36M^2}{R^2}\right)^{-1/2},$$

$$\frac{dl_4}{dR} = \frac{1}{2}\left(\frac{M}{R}\right)^{1/2}\frac{R-3M}{R-2M}\left(1 - \frac{2M}{R}\right)^{-1/4},$$