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Non-abelian gauge structure in neutrino mixing

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ABSTRACT

We discuss the existence of a non-abelian gauge structure associated with flavor mixing. In the specific case of two flavor mixing of Dirac neutrino fields, we show that this reformulation allows to define flavor neutrino states which preserve the Poincaré structure. Phenomenological consequences of our analysis are explored.

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1. Introduction

The study of flavor mixing and oscillations is of utmost importance in contemporary theoretical and experimental physics, especially in view of the recent experimental discovery of neutrino oscillations [1]. At a theoretical level, one important issue is the one of a correct definition of the flavor states, i.e., the ones describing oscillating neutrinos. In the standard quantum mechanical treatment, the well-known Pontecorvo states [2] are used and oscillation formulas are derived, which can describe efficiently the main aspects of such a phenomenon. However, it is clear that Pontecorvo states are only approximate since they are not eigenstates of the flavor neutrino charges. Thus Pontecorvo states lead to violation of the conservation of leptonic charge in the neutrino production vertices [3,4].

The solution to the above problem has been found in the context of Quantum Field Theory (QFT). Indeed, by considering mixing at level of fields, rather than postulating it as a property of states, unexpected features emerged [5]. It has been found that field mixing is associated with inequivalent representation of the canonical anticommutation relations, i.e., the vacuum for the mass eigenstates of neutrinos has been found to be unitarily inequivalent to the vacuum for the flavor eigenstates of neutrinos – the flavor vacuum. The non-perturbative vacuum structure associated with field mixing has been found to be a very general feature, independently of the nature of the fields [6–10]. It has also been shown that it leads to modifications of the flavor oscillation formulae [6,7,11,12].

In QFT flavor states can be straightforwardly defined as eigenstates of the flavor charges which are derived in a canonical way from the symmetry properties of the neutrino Lagrangian [13]. It has been shown that states defined in this way restore flavor charge conservation in weak interaction vertices, at tree level [14]. Moreover, such states turn out to be eigenstates of the momentum operator.

Despite the above mentioned results, the QFT treatment of flavor states still presents some open problems. One such issue is Lorentz invariance. Indeed, the flavor vacuum is not Lorentz invariant being explicitly time-dependent. As a consequence, flavor states cannot be interpreted in terms of irreducible representations of the Poincaré group. A possible way to recover Poincaré invariance for mixed fields has been explored in Refs. [15] where non-standard dispersion relations for the mixed particles have been related to non-linear realizations of the Poincaré group [16]. Another interesting issue concerns the invariance of the flavor oscillation formulas under Lorentz boosts [17].

The relation of neutrino masses and mixing with a possible violation of the Poincaré and *CPT* symmetries has been the subject of many efforts in the last decade [18]. A related line of research concerns the use of neutrino mixing and oscillations as a sensitive probe for quantum gravity effects, as quantum gravity induced decoherence is expected to affect neutrino oscillations [19]. Such effects have also been connected [20] to the non-trivial structure of the flavor vacuum introduced in [5].

In this Letter we propose a non-perturbative approach to the mixing of particles which overcomes the problems mentioned above. The basic idea is to view the mixing phenomenon as the

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result of the interaction of the neutrino fields with an external field, which as we shall see appears to be a non-abelian gauge field. This point of view allows to treat formally the mixed fields as free fields, avoiding in this way the problems with their interpretation in terms of the Poincaré group. The violation of relativistic invariance is now seen as a consequence of the presence of a fixed external field, which defines a preferred direction in space-time.

Our approach enables us to define flavor neutrino states which are simultaneous eigenstates of the flavor charges, of the momentum operators and of a new Hamiltonian operator for the mixed fields whose definition naturally emerges from our approach. This operator can be interpreted as the energy which can be extracted from flavor neutrinos through scattering. We discuss a possible test for our theoretical scheme, by looking at mixed neutrinos in the β decay, where the endpoint of the electron energy spectrum turns out to be different in our approach with respect to the standard prediction.

In the present Letter, we consider only the mixing of two Dirac fermion fields. Similar results hold also for the case of mixing of boson fields, and for the case of three flavors. An analysis of these instances will be presented elsewhere.

2. Two-flavor neutrino mixing

We begin with the Lagrangian density describing two mixed neutrino fields:

$$\mathcal{L} = \bar{\nu}_e(i\partial\!\!\!/ - m_e)\nu_e + \bar{\nu}_\mu(i\partial\!\!\!/ - m_\mu)\nu_\mu - m_{e\mu}(\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e). \quad (1)$$

The standard treatment of the problem is based on the observation that this Lagrangian, being quadratic, can be diagonalized by a canonical transformation of the field operators (called the *mixing transformation*):

$$\nu_e = \nu_1\cos\theta + \nu_2\sin\theta, \quad (2)$$

$$\nu_\mu = -\nu_1\sin\theta + \nu_2\cos\theta, \quad (3)$$

so that one simply gets the sum of two free Dirac Lagrangians:

$$\mathcal{L} = \bar{\nu}_1(i\partial\!\!\!/ - m_1)\nu_1 + \bar{\nu}_2(i\partial\!\!\!/ - m_2)\nu_2. \quad (4)$$

In the above equations, θ is the mixing angle and $m_e = m_1\cos^2\theta + m_2\sin^2\theta$, $m_\mu = m_1\sin^2\theta + m_2\cos^2\theta$, $m_{e\mu} = (m_2 - m_1)\sin\theta\cos\theta$.

From the above Lagrangian, one can derive the canonical energy–momentum tensor:

$$\begin{aligned} T_{\rho\sigma} &= \bar{\nu}_e i\gamma_\rho\partial_\sigma\nu_e - \eta_{\rho\sigma}\bar{\nu}_e\big(i\gamma^\lambda\partial_\lambda - m_e\big)\nu_e + \bar{\nu}_\mu i\gamma_\rho\partial_\sigma\nu_\mu \\ &\quad - \eta_{\rho\sigma}\bar{\nu}_\mu\big(i\gamma^\lambda\partial_\lambda - m_\mu\big)\nu_\mu + \eta_{\rho\sigma}m_{e\mu}(\bar{\nu}_e\nu_\mu + \bar{\nu}_\mu\nu_e) \\ &= \bar{\nu}_1 i\gamma_\rho\partial_\sigma\nu_1 - \eta_{\rho\sigma}\bar{\nu}_1\big(i\gamma^\lambda\partial_\lambda - m_1\big)\nu_1 + \bar{\nu}_2 i\gamma_\rho\partial_\sigma\nu_2 \\ &\quad - \eta_{\rho\sigma}\bar{\nu}_2\big(i\gamma^\lambda\partial_\lambda - m_2\big)\nu_2, \end{aligned} \quad (5)$$

where $\eta_{\rho\sigma} = \mathrm{diag}(+1, -1, -1, -1)$ is the Minkowskian metric tensor. From this tensor it follows the total Hamiltonian:

$$\begin{aligned} H &= \int d^3\mathbf{x}\, T^{00} \\ &= \int d^3\mathbf{x}\, \nu_1^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + \beta m_1)\nu_1 \\ &\quad + \int d^3\mathbf{x}\, \nu_2^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + \beta m_2)\nu_2, \end{aligned} \quad (6)$$

which is just the sum of two free field Hamiltonians: $H = H_1 + H_2$.

Analogously, a momentum operator is defined as:

$$P^i = \int d^3\mathbf{x}\, T^{0i} = i\int d^3\mathbf{x}\, \nu_1^\dagger\partial^i\nu_1 + i\int d^3\mathbf{x}\, \nu_2^\dagger\partial^i\nu_2, \quad i = 1, 2, 3, \quad (7)$$

which again is the sum of two free field contributions.

To the free fields ν_j there are associated two conserved (Noether) charges:

$$Q_j = \int d^3\mathbf{x}\, \nu_j^\dagger(x)\nu_j(x), \quad j = 1, 2, \quad (8)$$

with the total charge $Q = Q_1 + Q_2$. The analysis of symmetries of the Lagrangian in the flavor basis (1), leads to the identification of the (non-conserved) flavor charges [13]:

$$Q_\sigma(x_0) = \int d^3\mathbf{x}\, \nu_\sigma^\dagger(x)\nu_\sigma(x), \quad \sigma = e, \mu, \quad (9)$$

with $Q_e(x_0) + Q_\mu(x_0) = Q$. Flavor charges describe the phenomenon of neutrino oscillations, see Appendix A.

It is interesting to consider the relation between the two sets of charges:

$$Q_e(x_0) = \cos^2\theta\, Q_1 + \sin^2\theta\, Q_2 + \sin\theta\cos\theta\int d^3\mathbf{x}\big[\nu_1^\dagger(x)\nu_2(x) + \nu_2^\dagger(x)\nu_1(x)\big], \quad (10)$$

$$Q_\mu(x_0) = \sin^2\theta\, Q_1 + \cos^2\theta\, Q_2 - \sin\theta\cos\theta\int d^3\mathbf{x}\big[\nu_1^\dagger(x)\nu_2(x) + \nu_2^\dagger(x)\nu_1(x)\big]. \quad (11)$$

The appearance of terms that cannot be written in terms of Q_j is related to a non-trivial structure of the flavor Hilbert space [5], see Appendix A.

3. Flavor mixing as a non-abelian gauge theory

We now show that the Lagrangian (1) can be formally written as a non-abelian gauge theory. In the following we shall use the conventions of Ref. [21].

The starting point is the observation that the mixing interaction can be consistently viewed as the interaction of the flavor neutrino fields with a constant external gauge field. The most direct way of seeing this goes through the Euler–Lagrange equations corresponding to the Lagrangian (1), namely:

$$i\partial_0\nu_e = (-i\boldsymbol{\alpha}\cdot\nabla + \beta m_e)\nu_e + \beta m_{e\mu}\nu_\mu, \quad (12)$$

$$i\partial_0\nu_\mu = (-i\boldsymbol{\alpha}\cdot\nabla + \beta m_\mu)\nu_\mu + \beta m_{e\mu}\nu_e, \quad (13)$$

where α_i, $i = 1, 2, 3$, and β are the usual Dirac matrices in a given representation. Here we choose the following representation:

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}, \quad (14)$$

where σ_i are the Pauli matrices and $\mathbb{1}$ is the 2×2 identity matrix. The Euler–Lagrange equations can be compactly written as follows:

$$iD_0\nu_f = (-i\boldsymbol{\alpha}\cdot\nabla + \beta M_d)\nu_f, \quad (15)$$

where $\nu_f = (\nu_e, \nu_\mu)^T$ is the flavor doublet and $M_d = \mathrm{diag}(m_e, m_\mu)$ is a diagonal mass matrix. We have defined the (non-abelian) covariant derivative:

$$D_0 := \partial_0 + im_{e\mu}\beta\sigma_1, \quad (16)$$

where $m_{e\mu} = \frac{1}{2}\tan 2\theta\,\delta m$, and $\delta m := m_\mu - m_e$.

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