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Large N flavor β-functions: A recap

B. Holdom

Department of Physics, University of Toronto, Toronto, ON, Canada M5S1A7

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ABSTRACT

β-Functions for abelian and nonabelian gauge theories are studied in the regime where the large N flavor expansion is applicable. The first nontrivial order in the $1/N$ expansion is known for any value of $N\alpha$, and there are also various indications as to the nature of higher order effects. The singularity structure as a function of $N\alpha$ has implications for the existence of nontrivial fixed points.

For a sufficiently large number of flavors a nonabelian gauge theory will loose asymptotic freedom and will in this way resemble an abelian gauge theory. We shall thus focus our attention on the abelian case and then later extend the discussion to the nonabelian case.

It is generally believed that a $U(1)$ gauge theory with N charged fermions has a running coupling that grows monotonically towards the ultraviolet, and thus suffers from a Landau pole. This is the indication from the one-loop β-function. But there is much more known about the perturbative β-function and there have been recent calculations that have extended our knowledge to 5-loops. We also have complete knowledge of the first nontrivial order in the $1/N$ expansion for any value of $N\alpha$, and so this makes the large N expansion a useful way to organize the perturbative expansion. The question is whether any of this allows us to glean anything further about the possible existence of nontrivial fixed points. Although any perturbative approach will introduce a renormalization scheme dependence, it can still be hoped that the existence or nonexistence of a fixed point will leave some mark on the perturbative results.

According to a lattice result [1] there is no nontrivial fixed point in a $U(1)$ gauge theory for $N=4$. We shall be concerned with larger N where the large N expansion suggests other possibilities. This provides some motivation to study larger N values on the lattice as well, for sufficiently large values of $N\alpha$. Extending the current lattice result to $N=8,12,16,\ldots$ would appear to be relatively straightforward.

The $U(1)$ β-function is defined as

$$\beta(\alpha)=\frac{\partial\ln\alpha}{\partial\ln\mu}. \tag{1}$$

The one loop result is $\beta(\alpha)=2A/3$ where $A\equiv N\alpha/\pi$. We may write an expansion in $1/N$ as follows,

$$\frac{3}{2}\frac{\beta(\alpha)}{A}=1+\sum_{i=1}^{\infty}\frac{F_i(A)}{N^i}. \tag{2}$$

The "1" corresponds to the one loop result and we shall refer to it as the zeroth order term in the $1/N$ expansion. Each $F_i(A)$ represents a class of diagrams having the same dependence on N when A is held fixed, and such diagrams exist to all orders in A. If the functions $|F_i(A)|$ were bounded then for sufficiently large N one could conclude that the zeroth order term dominates and that the Landau pole is unavoidable. But singularities in the $F_i(A)$ will keep us from drawing this conclusion.

We collect together what is known about the $F_i(A)$'s in the $\overline{\text{MS}}$ renormalization scheme.

$$F_1(A)=\int_0^{\frac{A}{3}}I_1(x)\,dx, \tag{3}$$

$$I_1(x)=\frac{(1+x)(2x-1)^2(2x-3)^2\sin(\pi x)^3\Gamma(x-1)^2\Gamma(-2x)}{(x-2)\pi^3}, \tag{4}$$

E-mail address: bob.holdom@utoronto.ca.

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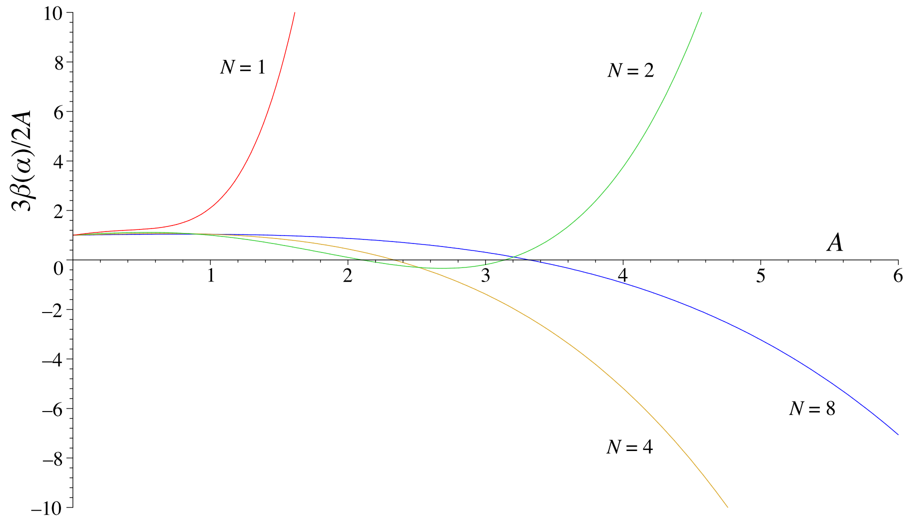

Fig. 1. The sum of the known contributions to $3\beta(\alpha)/2A$ in (3)–(7).

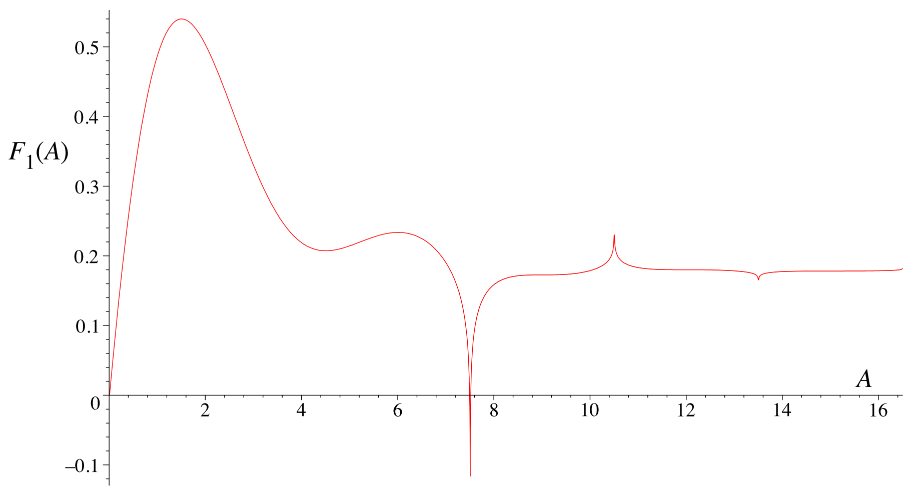

Fig. 2. $F_1(A)$ as defined in (3).

$$F_2(A) = -\frac{3}{32}A^2 + \left(\frac{95}{288} - \frac{13}{12}\zeta(3)\right)A^3 + \left(\frac{4961}{13824} + \frac{11\pi^4}{2880} - \frac{119\zeta(3)}{144}\right)A^4 + \cdots, \tag{5}$$

$$F_3(A) = -\frac{69}{128}A^3 + \cdots, \tag{6}$$

$$F_4(A) = \left(\frac{4157}{2048} + \frac{3}{8}\zeta(3)\right)A^4 + \cdots. \tag{7}$$

Important for our study is the fact that $F_1(A)$ is known completely [2]. We have expressed the integrand $I_1(x)$ in a form that makes more clear the location of its zeros and poles. The A^3 terms in $F_2(A)$ and $F_3(A)$ were calculated in [3], the A^4 term in $F_2(A)$ in [4], and the $F_4(A)$ term in [5]. The latter two results are 5-loop calculations.

One way to express the results in (3)–(7) is to plot their sum and ignore what is not known. The result for the $3\beta(\alpha)/2A$ for various N is displayed in Fig. 1. A zero would indicate a nontrivial fixed point, but the zeros are occurring at values of A that are too high to ignore higher order terms. Thus we cannot deduce much from this plot, except to notice sensitivity of the β-function to N.

The 2-loop contribution to $\beta(\alpha)$ involves one fermion loop and one internal photon and it gives rise to the first term in the expansion of $F_1(A)$, which is $\frac{3}{4}A$. The higher order terms in $F_1(A)$ correspond to the insertion of the appropriate number of fermion loops on the photon line, and these bubble chains have been summed up to produce the result (3) [2]. An important feature of $F_1(A)$ is that its expansion displays a finite radius of convergence, which is nonvanishing due to the slower than factorial growth in the number of diagrams. We expect this to be true of the other $F_i(A)$'s as well.

Simple poles of alternating sign appear in $I_1(x)$ at $x = \frac{5}{2} + n$ for integer $n \geqslant 0$. The integration can be handled with a Cauchy principal value prescription and the result is shown in Fig. 2. Clearly $F_1(A)$ only changes logarithmically as the singular points are approached. And these singularities become weaker for larger n. Close to the first singularity at $A = 15/2$ we find

$$F_1(A) \approx \frac{7}{15\pi^2}\Re\left(\log\left(1 - \frac{2}{15}A\right)\right) + 0.3056. \tag{8}$$

Nevertheless even this weak singularity can cause the β-function at this order to vanish at a fixed value of N. As Fig. 2 indicates there will be two nearly coincident zeros of $1 + F_1(A)/N$ at