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Dirac quantization of open p-brane

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Abstract

We give the scheme of Dirac quantization of open p-brane in the D-brane background. Treating the mixed boundary conditions as primary constraints, we get a set of secondary constraints, then the constraints conditions are shown to be equivalent to orbifold conditions imposed on normal p-brane modes.

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1. Introduction

In the picture of point-like particle, the finite mass or charge would lead to infinite density. Maybe the infinite self-energy of the point-like charge has something to do with it. To deal with such situations, physicists have developed lots of methods and programs, such as renormalization and finite size of electron and so on [1–4].

String theories have been regarded as the most promising candidates for unified theories of all fundamental interactions. As is well known, string theories achieve remarkable successes in answering some of the most vexing problems of high-energy physics [5–9]. Since strings are the extended objects in one dimension, it seems that they can avoid the infinite density, so as infinite self-energy, at least in a rough looking. By the motivation from point to string, the generalization to higher dimensional extended objects should be possible and natural. A generic element of this class is a p-dimensional spatially extended structure called the p-brane [5,10]. The p-brane sweeps out a $(p+1)$-dimensional world-volume in the embedding spacetime. After finding p-brane, the natural step would be to quantize them. Ref. [11] had successfully quantized the strings ending on the D-brane by decomposing the spacetime coordinates and momentums into normal string modes. By generalizing Ref. [10]'s researches, we have quantized general p-brane $(p = 2, 3, \ldots)$.

Our Letter is organized as follows: First we decompose the spacetime coordinates and momenta into normal p-brane modes and deduce the Hamiltonian of p-brane, we then go to $p = 2$ explicitly, and obtain all different Dirac brackets, and then we generalize the result to the general open p-brane, finally we close our Letter with summary and conclusion.

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2. p-brane action and Hamiltonian

The bosonic part of classical action for an open p-brane ending on a D_q-brane is given by [11]

$$I = -\frac{1}{4\pi\alpha'}\int_M d^{p+1}\xi\,\sqrt{-h}\left[G_{\mu\nu}h^{\alpha\beta}\frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X_\mu}{\partial\xi^\beta} + (p-1)\right], \tag{2.1}$$

where $\alpha, \beta = 0, 1, \ldots, p$ and $\mu = 0, 1, \ldots, d$, $(p-1)$ is the cosmological terms. For convenience and no losing general property, we consider the flat background, $G_{\mu\nu} = \eta_{\mu\nu}$ and set $2\pi\alpha' = 1$.

Choosing the metrics as $h_{\alpha\beta} = \eta_{\alpha\beta} = (-, +, +, \ldots, +)$. We find the Hamiltonian and the canonical momenta as

$$\mathcal{H} = \frac{1}{2}\left(P^i\right)^2 + \frac{1}{2}\left(P^a\right)^2 + \frac{1}{2}\sum_{k=1}^{p}\left(\partial_k X^\mu\right)^2 + (p-1), \tag{2.2}$$

$$P^\mu = \partial_\tau X^\mu, \tag{2.3}$$

where $\partial_k = \partial/\partial\sigma^k$, $i = 0, 1, 2, \ldots, q$, $a = q+1, \ldots, d$. In order to guarantee all the p boundary conditions are Neumann type, it is necessary for $q \geqslant p$. The Neumann boundary conditions imposed are

$$\partial_k X^i(0) = \partial_k X^i(\pi) = 0, \tag{2.4}$$

and the single D_q-brane is fixed by

$$X^a\left(\sigma^k = 0\right) = X^a\left(\sigma^k = \pi\right) = x^a. \tag{2.5}$$

We can always require that $x^a = 0$, i.e., the boundary conditions become

$$X^a\left(\sigma^k = 0\right) = X^a\left(\sigma^k = \pi\right) = 0. \tag{2.6}$$

The Hamiltonian for the free open p-brane is

$$\begin{aligned} H &= \frac{1}{2}\int\frac{d^p\xi}{(2\pi)^p}\left[\left(P^i\right)^2 + \left(P^a\right)^2 + \sum_{k=1}^{p}\left(\partial_k X^k\right)^2 + \sum_{k=1}^{p}\left(\partial_k X^a\right)^2 + (p-1)\right] \\ &= \frac{1}{2}\eta_{ij}\sum_{n_1 n_2\cdots n_p}\left[P^i_{n_1 n_2\cdots n_p}P^j_{(-n_1)(-n_2)\cdots(-n_p)} + \left(n_1^2 + \cdots n_p^2\right)X^i_{n_1 n_2\cdots n_p}X^j_{(-n_1)(-n_2)\cdots(-n_p)}\right] \\ &\quad + \frac{1}{2}\eta_{ab}\sum_{n_1 n_2\cdots n_p}\left[P^a_{n_1 n_2\cdots n_p}P^b_{(-n_1)(-n_2)\cdots(-n_p)} + \left(n_1^2 + \cdots n_p^2\right)X^a_{n_1 n_2\cdots n_p}X^b_{(-n_1)(-n_2)\cdots(-n_p)}\right] + \text{const}, \end{aligned} \tag{2.7}$$

where we have decomposed X^μ and P^μ as follows

$$\begin{aligned} X^\mu &= \sum_{n_1 n_2\cdots n_p} X^\mu_{n_1 n_2\cdots n_p} e^{in_1\sigma^1}e^{in_2\sigma^2}\cdots e^{in_p\sigma^p}, \\ P^\mu &= \sum_{n_1 n_2\cdots n_p} P^\mu_{n_1 n_2\cdots n_p} e^{-in_1\sigma^1}e^{-in_2\sigma^2}\cdots e^{-in_p\sigma^p}. \end{aligned} \tag{2.8}$$

The const (coming from the integral of $(p-1)$) will not give any contributions to the quantization of the open p-brane in the Dirac brackets, thus it can be neglected.

3. For $p = 2$ case

In this section, as an example, we mainly deal with the open 2-brane explicitly. In this case, the Hamiltonian can be written as

$$\begin{aligned} H &= \frac{1}{2}\sum_n\sum_m \eta_{ij}\left[P^i_{nm}P^j_{(-n)(-m)} + \left(n^2 + m^2\right)X^i_{nm}X^j_{(-n)(-m)}\right] \\ &\quad + \frac{1}{2}\sum_n\sum_m \eta_{ab}\left[P^a_{nm}P^b_{(-n)(-m)} + \left(n^2 + m^2\right)X^a_{nm}X^b_{(-n)(-m)}\right]. \end{aligned} \tag{3.1}$$

For simplicity, we firstly consider only X^i and P^i. The Neumann boundary conditions to be imposed on the ends of open 2-brane are $\partial_1 X^i(0) = \partial_1 X^i(\pi) = 0$, $\partial_2 X^i(0) = \partial_2 X^i(\pi) = 0$. In terms of normal 2-brane modes, these boundary conditions are rewritten