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Remarks on gravitational interaction in Kaluza–Klein models

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ABSTRACT

In these remarks, we clarify the problematic aspects of gravitational interaction in a weak-field limit of Kaluza–Klein models. We explain why some models meet the classical gravitational tests, while others do not. We show that variation of the total volume of the internal spaces generates the fifth force. This is the main reason of the problem. It happens for all considered models (linear with respect to the scalar curvature and nonlinear $f(R)$, with toroidal and spherical compactifications). We explicitly single out the contribution of the fifth force to nonrelativistic gravitational potentials. In the case of models with toroidal compactification, we demonstrate how tension (with and without effects of nonlinearity) of the gravitating source can fix the total volume of the internal space, resulting in the vanishing fifth force and consequently in agreement with the observations. It takes place for latent solitons, black strings and black branes. We also demonstrate a particular example where non-vanishing variations of the internal space volume do not contradict the gravitational experiments. In the case of spherical compactification, the fifth force is replaced by the Yukawa interaction for models with the stabilized internal space. For large Yukawa masses, the effect of this interaction is negligibly small, and considered models satisfy the gravitational tests at the same level of accuracy as general relativity.

1. Introduction

In our recent papers [1–7], we investigated classical gravitational tests (the perihelion shift, the deflection of light and the time delay of radar echoes) in Kaluza–Klein (KK) models. We paid attention mainly to models with toroidal compactification of extra dimensions, i.e. with compact and flat internal spaces. On the one hand, these theories are very popular in the literature devoted to KK models. On the other hand, it is well known that the gravitational tests in general relativity are calculated on the background of a flat metrics. This background metrics is perturbed by a pointlike mass. In a weak-field limit, such matter source has a dustlike equation of state. In general relativity, this formulation of the problem leads to formulas for gravitational tests, which are in good agreement with the experimental data [8]. Therefore, to generalize this approach, we also supposed that the background metrics (when the matter source is absent) is flat for our external four-dimensional spacetime and internal spaces, and a pointlike matter source has a dustlike equation of state in all spatial dimensions. To our surprise, this approach does not work in KK models. Obtained formulas strongly contradict the observations [1]. It is important to note that the result does not depend on the pointlike[1] approximation. Instead of the deltashaped form, we can consider a compact object in the form of a perfect fluid with the dustlike equation of state in all spatial dimensions, and we obtain the same negative result [3]. It turned out that to satisfy the experimental data, the matter source should have negative equations of state (tension) in the internal spaces [2,3]. For example, latent solitons have such tension and they satisfy the gravitational tests at the same level of accuracy as general relativity [3]. The uniform black strings and black branes are particular examples of the latent solitons. The similar situation takes place for nonlinear (with respect to the scalar curvature) KK theories with toroidal compactification [4,5]. Here, a pointlike mass with the dustlike equation of state in all spatial dimensions contradicts the observations [4], but there are two classes of asymptotic latent solitons, which are in agreement with the observations at the same level of accuracy as general relativity [5]. For both of these classes, a gravitating mass has tension in the internal space.

For black strings and black branes, the notion of tension is defined, e.g., in [10] and it follows from the first law for black

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[1] In the most of cases, when we use the word "pointlike", we usually mean a gravitating mass which has a deltashaped form in the external space and is uniformly smeared over the internal space. In this case, the nonrelativistic gravitational potential exactly coincides with the Newtonian one [9].

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hole spacetimes [11–13]. We study gravitational experiments for KK models in the Solar system. Obviously, our Sun is not a relativistic astrophysical object. Therefore, to calculate the motion of a test body in the vicinity of the Sun, there is no need to take into account the relativistic properties of black holes/strings (e.g., the presence of the horizon, a possible phase transition between black strings and black branes [14], etc.). It is sufficient to use the corresponding black strings/branes metrics in the weak-field limit. Additionally, it is worth noting that the metric coefficients for uniform black strings/branes depend only on the three-dimensional radius–vector. Therefore, a matter source is uniformly smeared over the extra dimensions and the nonrelativistic gravitational potential exactly coincides with the Newtonian one. Strictly speaking, to be in agreement with gravitational experiments, it is not sufficient to have the "correct" nonrelativistic gravitational potential. All models with the "smeared" extra dimensions have the Newtonian potential in the weak-field limit (see, e.g., the class of the exact solitonic solutions [3]). To achieve the concordance with the experimental data, the temporal and three-dimensional spatial metric coefficients should relate correspondingly with each other. The uniform black strings/branes have the right relation but the sources with the dustlike equation of state in all spatial dimensions do not have.

Then, we generalize our studies to the case of KK models with spherical compactification of the internal space [6,7]. Here, the background metrics is not flat because the internal space (e.g., the two-sphere) is curved. To create such background, we need to introduce a background matter source, e.g., in the form of a perfect fluid. A pointlike mass disturbs this background. We demonstrate that there are two different models with and without a bare cosmological constant. In the former case, a pointlike mass contradicts the observations [6]. However, in the latter case, the perturbed metric coefficients have the Yukawa type corrections with respect to the usual Newtonian gravitational potential [7]. These corrections are negligible in the Solar system. Therefore, the considered model satisfies the gravitational tests.

The main objective of this Letter is to explain why some models with toroidal and spherical compactifications fail with the observations, while the others are in good agreement with the experimental data. It is generally accepted that the variations of the internal space volume produce the fifth force which may lead to the contradiction with the observations. Therefore, first, we single out explicitly the contribution of such variations to nonrelativistic gravitational potentials. We show that the variation of the total volume of the internal spaces generates the fifth force for the models which fail with the observations. This is the main reason of the problem in these models. Then, we demonstrate that tension can fix the total volume of the internal space, eliminating the fifth force. Second, we ask the following question. Is a vanishing variation of the internal space volume a necessary and sufficient condition of the agreement with the gravitational tests? To our surprise, the answer is negative. We find a particular example where non-vanishing variations of the internal space volume do not lead to the contradiction with the gravitational experiments. Moreover, in this model, we can make the variations equal to zero, but then the gravitational tests fail here. We also consider briefly the model with a pointlike gravitating source in the case of spherical compactification of the internal space. Here, tension is absent and the fifth force is replaced by the Yukawa interaction. For sufficiently large Yukawa masses (it happens, e.g., in the Solar system), the effect of this interaction is negligibly small, and the considered model satisfies the gravitational tests at the same level of accuracy as general relativity.

The Letter is structured as follows. In Section 2, we consider linear models with toroidal compactification of the internal spaces and different types of gravitating sources. These models are generalized to nonlinear $f(R)$ ones in Section 3. In Section 4, we investigate models with spherical compactification of the internal space. The main results are summarized in the concluding Section 5. In Appendix A, we demonstrate that effective four-dimensional approach confirms the results obtained in Section 2.

2. Linear models with toroidal compactification

In this section we analyze linear with respect to the scalar curvature KK models with toroidal compactification of the internal spaces.

2.1. Pointlike mass

First, we investigate a model with a pointlike massive source. We consider a weak-field limit. It means that the gravitational field is weak, i.e. the metrics is only slightly perturbed from its flat spacetime value:

$$g_{ik} \approx \eta_{ik} + h_{ik}. \tag{1}$$

Here, the metric perturbations $h_{ik} \sim O(1/c^2)$, where c is the speed of light, $i,k = 0,1,\dots,D$, and D is a total number of the spatial dimensions. In the weak-field limit, the only nonzero component of the energy–momentum tensor for a pointlike mass at rest is $T_{00} \approx \rho_D c^2 \sim O(c^2)$. ρ_D is a D-dimensional rest mass density, and for a pointlike mass m we have $\rho_D = m\delta(\mathbf{r}_D)$. Usually, we deal with the case of matter sources, which are uniformly smeared over the extra dimensions [9]. In this case, the metric coefficients may depend only on coordinates of the external space.[2] For the smeared extra dimensions, the nonrelativistic three-dimensional mass density ρ_3 is connected with the D-dimensional one as follows: $\rho_D = \rho_3/V_{D'} = m\delta(\mathbf{r}_3)/V_{D'}$, where $D' = D-3$ is a total number of the extra dimensions and $V_{D'}$ is a total volume of the unperturbed internal spaces. For example, if a_i are periods of tori, then $V_{D'} = \prod_{i=1}^{D'} a_i$. For such setup, the Einstein equation

$$R_{ik} = \frac{2S_D\tilde{G}_{\mathcal{D}}}{c^4}\left(T_{ik} - \frac{1}{D-1}g_{ik}T\right), \tag{2}$$

where $S_D = 2\pi^{D/2}/\Gamma(D/2)$ is a total solid angle (a surface area of the $(D-1)$-dimensional sphere of the unit radius) and $\tilde{G}_{\mathcal{D}}$ is the gravitational constant in the $(\mathcal{D} = D+1)$-dimensional spacetime, is reduced to a system of linearized equations with the following nonzero solutions [1]:

$$h_{00} = -\frac{2(D-2)}{D-1}\frac{2G_N m}{c^2 r_3}, \tag{3}$$

$$h_{\alpha\alpha} = -\frac{2}{D-1}\frac{2G_N m}{c^2 r_3}, \quad \alpha = 1,2,3, \tag{4}$$

$$h_{\mu\mu} = -\frac{2}{D-1}\frac{2G_N m}{c^2 r_3}, \quad \mu = 4,5,\dots,D, \tag{5}$$

where we introduce Newton's gravitational constant

$$G_N = \frac{S_D}{4\pi}\frac{\tilde{G}_{\mathcal{D}}}{V_{D'}}. \tag{6}$$

Hereafter, the Latin indices $i,k = 0,\dots,D$, the Greek indices $\alpha,\beta = 1,2,3$ and the Greek indices $\mu,\nu = 4,5,\dots,D$.

[2] For the smeared extra dimensions, KK modes are absent. Our following analysis can be easily generalized to the case of non-smeared extra dimensions. In this case, KK modes are present in considered models. However, in the Solar system, they are negligible compared with the zero mode [1], and such generalization does not change the main conclusions of our Letter.