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CDT meets Hořava–Lifshitz gravity

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ABSTRACT

The theory of causal dynamical triangulations (CDT) attempts to define a nonperturbative theory of quantum gravity as a sum over spacetime geometries. One of the ingredients of the CDT framework is a global time foliation, which also plays a central role in the quantum gravity theory recently formulated by Hořava. We show that the phase diagram of CDT bears a striking resemblance with the generic Lifshitz phase diagram appealed to by Hořava. We argue that CDT might provide a unifying nonperturbative framework for anisotropic as well as isotropic theories of quantum gravity.

1. Introduction

A major unsolved problem in theoretical physics is to reconcile the classical theory of general relativity with quantum mechanics. Recently there has been a resurgence in interest in using mundane quantum field theory to address this question. Progress over the last ten years in the use of renormalization group (RG) techniques [1] suggests that the so-called asymptotic safety scenario, originally put forward by S. Weinberg [2], may be realized, namely, the existence of a nontrivial ultraviolet fixed point, where one can define a theory of quantum gravity.

In tandem with this approach, the method of Causal Dynamical Triangulation (CDT) has been developed, likewise with the aim of defining and constructing a nonperturbative quantum gravity theory [3–7] (for recent reviews, see [8]). CDT provides a lattice framework in which a variety of nonperturbative field-theoretical aspects of quantum gravity can be studied, including in principle predictions from other candidate theories. Despite the fact that the CDT and the RG approaches use rather different sets of tools, they might be two sides of the same coin. Locating a suitable UV fixed point in causal dynamical triangulations would provide strong evidence that this is indeed the case and that "asymptotic safety" is on the right track.

More recently, P. Hořava has suggested yet another field-theoretical approach to quantum gravity in the continuum [9], since dubbed Hořava–Lifshitz gravity, where the four-dimensional diffeomorphism symmetry of general relativity is explicitly broken. Assuming a global time-foliation, time and space are treated differently, in the sense that only suitable *second*-order derivatives in time appear to render the quantum theory unitary, while higher-order spatial derivatives ensure renormalizability.

A common key ingredient in both CDT and Hořava–Lifshitz gravity is a global time foliation, with the difference that in CDT this is not directly associated with a violation of diffeomorphism symmetry, since the dynamics is defined directly on the quotient space of metrics modulo diffeomorphisms. This raises the question whether new insights can be gained by analyzing and interpreting CDT quantum gravity in a generalized, anisotropic framework along the lines of Hořava–Lifshitz gravity. The reference frame until now has been a covariant one, assuming that any UV fixed point found in the CDT formulation could be identified with that found in the covariant renormalization group approach, appealing to the general sparseness of fixed points.[1] At the same time, we have presented general arguments in favour of a reflection-positive transfer matrix in the (Euclideanized version of) CDT [10,11]. Thus the conditions for a unitary quantum field theory at the UV fixed point are also met. The philosophy behind formulating gravity at a Lifshitz point was that unitarity in a theory of quantum gravity should be

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[1] Of course, one should also show that a lattice fixed point reproduces the critical exponents of the RG treatment.

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the prime requirement, rather than treating space and time on the (almost) equal footing required by special relativity. We conclude that the CDT approach not only shares the time-foliated structure of spacetime, but also the enforcement of unitarity by construction with Hořava–Lifshitz gravity.

This led us to asking whether CDT may be able to capture aspects of the latter, despite the fact that no higher-order spatial derivative terms are put in by hand in the CDT action. Some support for this idea comes from the fact that one UV result which can be compared explicitly, namely, the nontrivial value of the spectral dimension of quantum spacetime, appears to coincide in both approaches [12,13]. Interestingly, also the renormalization group approach was able to reproduce the same finding, after the spectral dimension had first been measured in simulations of CDT quantum gravity, a result taken at the time as possible corroboration of the equivalence between the CDT and RG approaches [14].[2]

In view of the considerations outlined above, we have returned to a closer analysis of the basic CDT phase diagram. In what follows, we will report on some striking similarities between the phase diagram of causal dynamically triangulated gravity and the Lifshitz phase diagram promoted in Hořava–Lifshitz gravity. They become apparent when one identifies "average geometry", presumably related to the conformal mode of the geometry in some way, with the order parameter ϕ of an effective Lifshitz theory. We find that the phase structure allows potentially for both an anisotropic and an isotropic UV fixed point, opening the exciting prospect that CDT can serve as a nonperturbative lattice foundation for both the renormalization group approach and Hořava–Lifshitz gravity, in the same way as theories on fixed lattices provide us with nonperturbative definitions of quantum field theories in the formulation of K. Wilson.

2. Causal dynamical triangulation

We will merely sketch the setup used in CDT, and refer to [11, 4,7] for more complete descriptions and to [16] for the rationale behind the formulation. We attempt to define the path integral of quantum gravity by summing over a class of piecewise linear spacetime geometries, much in the same way as one can define the path integral in ordinary quantum mechanics by dividing the time into intervals of length a, considering paths which are linear between $t_n = na$ and $t_{n+1} = (n+1)a$, and then taking the limit of vanishing "lattice spacing", $a \to 0$.

Let us introduce a time slicing labeled by discrete lattice times t_n. The spatial hypersurface labeled by t_n has the topology of S^3 and is a piecewise flat triangulation, obtained by gluing together identical, equilateral tetrahedra with link lengt a_s, to be identified with the short-distance lattice cut-off. We now connect the three-dimensional triangulation of S^3 at t_n with that at time t_{n+1} by means of four types of four-simplices: four-simplices of type $(4,1)$, which share four vertices (in fact, an entire tetrahedron) with the spatial hypersurface at t_n and one vertex with the hypersurface at t_{n+1}; four-simplices of type $(1,4)$, where the roles of t_n and t_{n+1} are interchanged; four-simplices of type $(3,2)$, which share three vertices (in fact, an entire triangle) with the hypersurface at t_n, and two vertices with the hypersurface at t_{n+1} (belonging to the same spatial link); lastly, four-simplices of type $(2,3)$, defined analogously but with t_n and t_{n+1} interchanged.

These four-simplices have a number of links (and corresponding triangles and tetrahedra) connecting vertices in hypersurfaces t_n and t_{n+1}. We take all of these links to be time-like with (squared) length $a_t^2 = \alpha a_s^2$. The four-simplices are glued together such that the "slab" between hypersurfaces labeled by t_n and t_{n+1} has the topology $S^3 \times [0,1]$. We *say* that the hypersurfaces are separated by a proper distance $\sqrt{\alpha}a_t$, but this is not strictly speaking true if one takes the piecewise flat geometries (despite their curvature singularities) seriously as classical spacetimes. However, what is true is that all links connecting neighbouring hypersurfaces have proper length $\sqrt{\alpha}a_t$.

In the path integral we sum over all geometrically distinct piecewise linear geometries of this type, and with a fixed number of time steps. As an action we use the Einstein–Hilbert action, which has a natural realization on piecewise linear geometries, first introduced by Regge. The geometries allow a rotation to Euclidean geometries simply by rotating $\alpha \to -\alpha$ in the lower-half complex plane. The action changes accordingly and becomes the Euclidean Einstein–Hilbert Regge action of the thus "Wick-rotated" piecewise flat Euclidean spacetime. Its functional form becomes extremely simple because we use only two different kinds of building blocks, which contribute in discrete units to the four-volume and the scalar curvature. In this way the Euclidean action becomes a function of "counting building blocks", namely,

$$
\begin{aligned}
S_E &= \frac{1}{G}\int \sqrt{g}(-R+2\Lambda) \\
&\to -(\kappa_0 + 6\Delta)N_0 + \kappa_4\big(N_4^{(4,1)} + N_4^{(3,2)}\big) \\
&\quad + \Delta\big(2N_4^{(4,1)} + N_4^{(3,2)}\big),
\end{aligned}
\tag{1}
$$

where N_0 is the number of vertices, $N_4^{(4,1)}$ the number of four-simplices of type $(4,1)$ or $(1,4)$, and $N_4^{(3,2)}$ the number of four-simplices of type $(3,2)$ or $(2,3)$ in the given triangulated spacetime history. For later use we denote the total number $N_4^{(4,1)} + N_4^{(3,2)}$ of four-simplices by N_4. Furthermore, the parameter κ_0 in (1) is proportional to the inverse bare gravitational coupling constant, while κ_4 is related to the bare cosmological coupling constant. Finally, Δ is an asymmetry parameter which in a convenient way encodes the dependence of the action on the relative time–space scaling α introduced above, and is handy when studying the relation to Hořava–Lifshitz gravity. Vanishing $\Delta = 0$ implies $\alpha = 1$, and increasing Δ away from zero corresponds to decreasing α, i.e. the time-like links shrink in length when Δ is increased.

The rotation to Euclidean space is necessary in order to use Monte Carlo simulations as a tool to explore the theory nonperturbatively. For simulation-technical reasons it is preferable to keep the total number N_4 of four-simplices fixed during a Monte Carlo simulation, which implies that κ_4 effectively does not appear as a coupling constant. Instead we can perform simulations for different four-volumes if needed. To summarize, we are dealing with a statistical system of fluctuating four-geometry, whose phase diagram as function of the two bare coupling constants κ_0 and Δ we are going to explore next.

3. The CDT phase diagram

The CDT phase diagram was described qualitatively as part of the first comprehensive study of four-dimensional CDT quantum gravity [4]. For the first time, we are presenting here the real phase diagram (Fig. 1), based on computer simulations with $N_4 = 80\,000$. Because there are residual finite-size effects for universes of this size, one can still expect minor changes in the location of the transition lines as $N_4 \to \infty$. The dotted lines in Fig. 1 represent mere extrapolations, and lie in a region of phase space which is difficult to access due to inefficiencies of our computer algorithms.

[2] Inspired by the seemingly universal value of the UV spectral dimension, more general arguments about the underlying UV nature of spacetime have been put forward [15].