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Thermal fluctuations in viscous cosmology

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ABSTRACT

In this Letter we investigate the power spectrum of thermal fluctuations in very early stage of viscous cosmology. When the state parameter as well as the viscous coefficient of a barotropic fluid is properly chosen, a scale invariant spectrum with large non-Gaussianity can be obtained. In contrast to the results previously obtained in string gas cosmology and holographic cosmology, we find the non-Gaussianity in this context can be k-independent such that it is not suppressed at large scale, which is expected to be testified in future observation.

1. Introduction

Recent observed Cosmic Microwave Background (CMB) anisotropy [1–3] and the large scale structure of the universe [4–8] can be viewed as the results of the primordial density perturbations, which is characterized by a nearly scale invariant and Gaussian power spectrum [9]. Though at present completely understanding the origin of the primordial density perturbations is still an open question, one traditional point of view is thinking of quantum fluctuations in vacuum as the seed of such classical perturbations. Nevertheless, there is an alternative conjecture arguing that it may be due to thermal fluctuations of matter sources during the inflationary stage [10–13]. Unfortunately, thermal fluctuations in standard inflation models always generate a power spectrum with spectral index $n_s = 4$ [14,15]. Recently progress has been made to overcome this difficulty. People show that once some new physics is introduced, for instance in the context of the noncommutative inflation, holographic cosmology and loop cosmology, the scale invariant spectrum can be implemented in the thermal scenario as well [11,14–20].

Recently the thermal origin of the primordial density perturbation has received more attention since the non-Gaussianity of CMB has been further disclosed in the latest observation data. It indicates that at 95% confidence level, the primordial non-Gaussianity parameters for the local and equilateral models are in the region $-9 < f_{\rm NL}^{local} < 111$ and $-151 < f_{\rm NL}^{equil} < 253$, respectively [9]. If this result is confirmed by future experiments such as the Planck satellite, then it will be a great challenge to many slow-roll inflation models since the non-Gaussianity has to be greatly suppressed at $|f_{\rm NL}| < 1$ in most of these models [21,22]. Contrasting to fluctuations originated from vacuum, the fluctuations in thermal scenario are always not strictly Gaussian. This may provide an alternative way to seek the observable non-Gaussianity. Therefore, the thermal non-Gaussianity has been widely discussed in recent literatures [23–34].

In this Letter we intend to investigate the power spectrum of thermal fluctuations in the very early stage of viscous cosmology, where the matter source is a viscous barotropic fluid (the state parameter $w = \frac{p}{\rho} \in [-1, 1]$ is a constant). Since the viscous effect has an impact on the perturbation modes after they cross the thermal horizon, it is expected that the power spectrum should be different from the previous results even without other new mechanism or extra structure introduced. As a matter of fact, we find that provided the state parameter as well as the viscous coefficient of a barotropic fluid is properly chosen, a scale invariant spectrum with large non-Gaussianity can be obtained in this framework indeed.

The outline of our Letter is the following. In next section we present a brief review on the thermal fluctuations in the FRW universe filled with a perfect fluid and demonstrate the difficulty of gaining a scale-invariant spectrum in this model. Then in Section 3 we show that this difficulty can be overcome by introducing a viscous fluid and properly choosing its viscous coefficient. Its non-Gaussianity is investigated in Section 4. Finally the comparison with other cosmological models with thermal fluctuations is given in Section 5.

2. A review on thermal fluctuations in the FRW universe with a perfect fluid

In this section, we briefly review the thermal fluctuations in very early universe and derive its power spectrum based on ther-

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modynamical consideration. First of all, we consider the FRW universe filled with a perfect fluid with state parameter w. For a perfect fluid, the thermal system can reach equilibrium state through interactions at sub-horizon scales. In the thermal scenario of cosmological fluctuations, this process is called thermalization. Since during this process both of the extensive internal energy U and entropy S are well defined for a global equilibrium state, we have the following form of the first law of thermodynamics

$$dU = T\,dS - p\,dV. \tag{1}$$

For barotropic fluid, one finds that the relation between the energy density and the temperature is uniquely fixed by this thermodynamical law [14,15]

$$\rho = AT^m, \tag{2}$$

where $m = 1 + \frac{1}{w}$ and A is an integral constant. For radiation with $w = \frac{1}{3}$, this equation is nothing but the famous Stefan–Boltzmann law. On the other hand, the partition function of the thermal fluid is defined as

$$Z = \sum_r e^{-\beta E_r}, \tag{3}$$

where $\beta = T^{-1}$. The internal energy U inside a volume V is given by

$$U = \langle E\rangle = \frac{\sum_r E_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}} = -\frac{d\log Z}{d\beta}. \tag{4}$$

Therefore, the two-point correlation function for the density fluctuation $\delta\rho \equiv \rho - \langle\rho\rangle$ can be obtained as

$$\begin{aligned}\langle\delta\rho^2\rangle &= \frac{\langle\delta E^2\rangle}{V^2} = \frac{\langle E^2\rangle - \langle E\rangle^2}{V^2} = \frac{1}{V^2}\frac{d^2\log Z}{d\beta^2}\\ &= -\frac{1}{V^2}\frac{dU}{d\beta} = \frac{T^2 C_V}{R^6},\end{aligned} \tag{5}$$

where $C_V = (\frac{\partial U}{\partial T})_V = V\frac{d\rho}{dT} \equiv V\rho'$ is the specific heat and $R \sim V^{\frac{1}{3}}$ is the size of the thermal horizon. Thermal fluctuations generated from the matter inside R can be described by the thermodynamics above. But when thermal modes are pushed outside the horizon, they are frozen and become non-thermal governed by the theory of perturbations. Next we calculate the power spectrum of perturbations. Following most of literatures we identify the thermal horizon R with the Hubble horizon H^{-1} [14,15,18,19,25–27], which means our calculation of spectrum is always taken at the Hubble scale.

If perturbations are deeply in the horizon, the 0–0 component of the perturbative Einstein equation will reduce to the Poisson equation which relates the curvature fluctuations Φ_k and the density perturbations $\delta\rho_k$ as [35]

$$k^2\Phi_k = 4\pi G a^2 \delta\rho_k, \tag{6}$$

where $\delta\rho_k = k^{-\frac{3}{2}}\delta\rho$. Thus the power spectrum can be obtained by combining Eqs. (5) and (6)

$$\mathcal{P}_\Phi(k) \equiv \frac{k^3}{2\pi}\langle\Phi_k^2\rangle \sim \frac{a}{k}T^2\rho', \tag{7}$$

where we have used the condition $R = H^{-1} = \frac{a}{k}$. When the Φ modes leave the horizon, their amplitudes get fixed at whatever thermal amplitudes they have at crossing $k = aH$. For simplicity, we consider the spatially flat universe. Then using the Friedmann equation $H^2 \propto \rho$, we have

$$\mathcal{P}_\Phi(k) \sim \left[\frac{T^2\rho'}{\sqrt{\rho}}\right]_{k=aH}. \tag{8}$$

For a constant w, substituting the thermodynamical relation (2) into above equation leads to

$$\frac{d\ln\mathcal{P}_\Phi}{d\ln T} = 1 + \frac{m}{2}. \tag{9}$$

Furthermore, from the conservation equation of the fluid one has $\rho \propto a^{-3(1+w)}$ such that

$$a \propto T^{\frac{-m}{3(1+w)}}. \tag{10}$$

Thus we have

$$\frac{d\ln k}{d\ln T} = \frac{m(1+3w)}{6(1+w)}, \tag{11}$$

where we have used $k = aH \propto a\sqrt{\rho}$. Using (9) and (11), it is straightforward to calculate the spectral index as

$$n_s - 1 = \frac{d\ln\mathcal{P}_\Phi}{d\ln k} = \frac{d\ln\mathcal{P}_\Phi}{d\ln T}\frac{d\ln T}{d\ln k} = 3\frac{2+m}{m}\frac{w+1}{3w+1} = 3. \tag{12}$$

In this equation the relation $m = 1 + \frac{1}{w}$ has been applied. Therefore the spectrum is always blue and independent of the value of w. Of course this "*no-go result*" [15] is not consistent with the current experiments, in which n_s is restricted at $n_s = 0.960^{+0.014}_{-0.013}$ [9]. Then, to obtain a scale invariant spectrum in thermal scenario, one need introduce new physics to either relax some constraints due to thermodynamics, for instance as presented in [14,19], or modify the standard cosmological equations, as presented in [15]. Different from above considerations, in next section we would like to argue that the *no-go result* above can also be avoided if the matter source is a viscous fluid rather than a perfect one.

3. Realization of scale-invariant fluctuations in viscous cosmology

Viscous cosmology has been widely applied to investigate the structure and the evolution of the universe [36–50] (for recent review we refer to [51]). In this context the energy–momentum tensor of the fluid is given by

$$T_{\mu\nu} = (\rho + p - 3\xi H)u_\mu u_\nu + (p - 3\xi H)g_{\mu\nu}, \tag{13}$$

where the bulk viscosity coefficient $\xi = \xi(\rho)$ is usually a function of the energy density ρ of the fluid. Moreover, as pointed out in [52], ξ should be positive if the second law of thermodynamics is respected. In FRW universe the conservation equation $\nabla^\mu T_{\mu\nu} = 0$ becomes

$$\dot\rho + 3H(\rho + p - 3\xi H) = 0. \tag{14}$$

In this Letter, we choose a special kind of viscous fluids with a bulk viscosity coefficient $\xi(\rho) \propto \rho^{\frac{1}{2}}$, which has already been investigated in some literatures [39,43,53]. In this case, $\frac{3\xi H}{\rho} \equiv \alpha$ is a positive constant which is relevant to the viscosity of the fluid. Then given a constant w, we have a relation between the energy density ρ and the scale factor from the equation above

$$\rho \propto a^{-3(1+w-\alpha)}. \tag{15}$$

Obviously, the evolution of the universe depends not only on the state parameter w but also on α.

Now we discuss the thermalization of the matter source. Usually, the thermal scenario of cosmological fluctuations is established on the thermodynamics of the equilibrium state. Unlike the case with a perfect fluid, the viscosity brings dissipative effect such that the global description of the first law of thermodynamics in Eq. (1) no longer stands in general. Nevertheless, each particle in a viscous fluid satisfies the Gibbs relation in a local equilibrium state

$$T\,ds = d\frac{\rho}{n} + pd\frac{1}{n}, \tag{16}$$