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Acceleration of particles by black holes as a result of deceleration: Ultimate manifestation of kinematic nature of BSW effect

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ABSTRACT

The recently discovered so-called BSW effect consists in the unbound growth of the energy $E_{c.m.}$ in the center of mass frame of two colliding particles near the black hole horizon. We consider a new type of the corresponding scenario when one of two particles ("critical") remains at rest near the horizon of the charged near-extremal black hole due to balance between the attractive and repulsion forces. The other one hits it with a speed close to that of light. This scenario shows in a most pronounced way the kinematic nature of the BSW effect. In the extremal limit, one would gain formally infinite $E_{c.m.}$ but this does not happen since it would have require the critical massive particle to remain at rest on the null horizon surface that is impossible. We also discuss the BSW effect in the metric of the extremal Reissner–Nordström black hole when the critical particle remains at rest near the horizon.

1. Introduction

The recent finding of the effect of the unbound growth of the energy $E_{c.m.}$ in the center of mass frame due to collisions of particles near the black hole horizon [1] (BSW effect) attracts now much attention. Both manifestations of this effect in different situation are being studied in detail and also the very nature of the effect itself is under investigation. It was observed in [2] that the underlying physical reason of the BSW effect can be explained in kinematic terms. Namely, it turns out that, roughly speaking, a rapid particle collides with a slow one near the horizon, this leads to the growth of the relative velocity and, as a result, to the unbound growth of the corresponding Lorentz gamma factor, so the energy $E_{c.m.}$ becomes unbound near the horizon. This general circumstance was also confirmed in thorough analysis of the BSW effect in the Kerr metric [3]. Nonetheless, some doubts remain concerning the possibility to give an alternative explanation. If something is being accelerated to unbound energies, one is tempted to ask, what source does this, and what is the "physical" underlying reason of such an effect.

The aim of the present work is to reveal the kinematic nature of the BSW effect in the most pronounced way. To this end, we consider the situation when one of two colliding particles is motionless while the other one moves (as usual) with a finite energy in the frame of a distant observer. In a sense, this is the ultimate and clear manifestation of the kinematic nature of the effect under discussion that does not require to search for further hidden dynamic factors. The model which we discuss shows the key issue as clear as possible: the role of gravitation in producing the BSW effect of the unbound growth of $E_{c.m.}$ ("acceleration of particles") consists not in acceleration but in *deceleration* of one of two particles (in the sense that its velocity is reduced to zero)!

To achieve our goal, we consider the spherically symmetric metric of a charged black hole that admits the equilibrium of a particle that remains motionless. In other words, we want to balance the gravitation force by electrical repulsion. Apart from this, it is important that such a point be located in the vicinity of the horizon. For definiteness, we consider the innermost stable equilibrium point which is the counterpart of the innermost stable circular orbit for the Kerr metric [4]. Such orbits were discussed recently due to their potential astrophysical significance [5,6]. (See their generalization to "dirty" rotating black holes [7].) There exists also their analog in the magnetic field where the BSW effect was studied recently in [8].

The simplest choice would seem to be the Reissner–Nordström (RN) black hole but for this metric the "orbit" with the required properties exists for indifferent equilibrium only (see Section 5 below). Therefore, for the analog of inner stable orbits we take the charged black hole with nonzero cosmological constant Λ. It turns out that it is required that $\Lambda < 0$, so we deal mainly with the Reissner–Nordström–anti-de Sitter one (RN–AdS) which is sufficient for our purposes. It is also worth noting that interest to black holes with the cosmological constant $\Lambda < 0$ revived in recent years due to AdS/CFT correspondence [9]. In addition, we consider also another type of "orbit" – a particle in the state of indifferent equilibrium in the metric of the extremal Reissner–Nordström black hole.

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2. Equations of motion

Consider the space–time describing a charged black hole with the cosmological constant. Its metric can be written as

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2\,d\Omega^2, \tag{1}$$

$$f = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}. \tag{2}$$

Throughout the Letter we assume that the fundamental constants $G = c = \hbar = 1$. The horizon lies at $r = r_+$ where $f(r_+) = 0$. The electric potential

$$\varphi = \frac{Q}{r} + C \tag{3}$$

where the C is the constant of integration. It is assumed that we work in the gauge where the only nonvanishing component $A_0 = -\varphi$. For the asymptotically flat case, say, for the Reissner–Nordström or Kerr–Newman metric, it is usually chosen $C = 0$ to have $\varphi = 0$ at infinity (see, e.g., Eq. (3.63) of [10]). In the absence of asymptotic flatness, its choice becomes conditional. It is worth stressing that physically relevant quantities contain not the potential itself but the difference with respect to some reference point (infinity or horizon). For example, in black hole thermodynamics, the potential enters the action in the form $\frac{\varphi(r)-\varphi(r_+)}{\sqrt{f}}$ that is nonsingular at the horizon (see Eq. (4.15) of [11]). In equations of motion (see below) only the combination $E - q\varphi$ appears where q is the particle's charge. If we change the potential according to $\varphi \to \varphi + C$, the corresponding shift in the energy $E \to E + qC$. For convenience, we choose $C = 0$ in (3).

We restrict ourselves by radial motion since this case is the most interesting in the context under discussion. As is known, under the presence of the electromagnetic field, dynamics of the system is described by the generalized momentum P_μ related to the kinematic one $p_\mu = mu^\mu$ by the relation $p_\mu = P_\mu - qA_\mu$ where $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity of a test massive particle, τ is the proper time, A_μ is the vector potential. Due to staticity, the energy $E = -P_0$ of a particle moving in this metric is conserved, P_0 is the time component of the generalized momentum P_μ. Then, using also the relation $u^0 = g^{00}u_0$, we obtain (dot denotes the derivative with respect to the proper time τ)

$$\dot t = u^0 = \frac{X}{mf}, \tag{4}$$

$$X = E - q\varphi. \tag{5}$$

We assume that $\dot t > 0$, so that $E - q\varphi > 0$.

$$m^2\dot r^2 = -V_{eff} = X^2 - m^2 f. \tag{6}$$

Now, we are interested in equilibrium solutions $r = r_0 = const$,

$$V_{eff}(r_0) = 0. \tag{7}$$

Additionally, we require that they possess the following properties: (i) r_0 is a perpetual turning point, (ii) it lies near the horizon, $r_+ \to r_0$. Condition (i) means that, in addition to (7), equation

$$V'_{eff}(r_0) = 0 \tag{8}$$

should hold. Eqs. (7), (8) ensure that not only $\dot r$ but also all higher derivatives vanish.

It follows from (6), (7) that for a particle with $\dot r = 0$,

$$X(r_0) = m\sqrt{f(r_0)}. \tag{9}$$

It is instructive to elucidate for which types of black holes equations (7) and (8) are self-consistent near the horizon, so that equilibrium points exist there in agreement with requirement (ii). Physical motivation for considering this requirement comes from our main goal – investigation of the BSW effect since this effect occurs just in the vicinity of the horizon.

If we take the derivative of the effective potential V_{eff} in Eq. (6) and take into account also the relation (7), we obtain

$$-\frac{1}{2}V'_{eff}(r_0) = m\sqrt{f(r_0)}\frac{qQ}{r_0^2} - \frac{m^2}{2}f'(r_0). \tag{10}$$

Let us consider the limit $r_0 \to r_+$, so $f(r_0) \to 0$. Then, it follows from (10) that $V'_{eff}(r_0) \to -m^2\kappa$ where we used the fact for the metric (1) $\kappa = \frac{1}{2}f'(r_+)$. Thus if $\kappa \neq 0$, Eq. (8) cannot be satisfied in the horizon limit. Therefore, for nonextremal black holes the equilibrium points cannot exist near the horizon (although they can exist elsewhere at a finite distance from the horizon). This generalizes previous observations [3,7] made for rotating black holes. However, if $\kappa \to 0$, the equilibrium points close to the horizon do exist as will be shown below.

3. Properties of equilibrium point

For the Kerr metric [4] and, in general, for axially-symmetric rotating black holes [7], there are so-called innermost stable orbits (ISCO) which correspond to the threshold of stability. We consider now their analogs in our case, so we must add to (7) and (10), also equation

$$V''_{eff}(r_0) = 0. \tag{11}$$

For brevity, we will call this an innermost stable equilibrium point (ISEP).

We are interested in the near-horizon region where we can expand f in the Taylor series with respect to $x = r_0 - r_+$:

$$f = 2\kappa x + Dx^2 + Cx^3 \cdots. \tag{12}$$

From now on, we assume that κ is a small parameter, so a black hole is a near-extremal. Then, this leads to an interplay between two small quantities κ and x. We assume the condition

$$\kappa \ll Dx \tag{13}$$

which one can check a posteriori that for the solutions obtained.

Then, the procedure for the description of the equilibrium points is mathematically similar to that for the description of circular orbits in the background of rotating black holes [7]. In both cases, we are interested in solutions for which $\dot r = 0$ and which are on the threshold of stability. Therefore, I omit technical details (which are connected with simple but rather cumbersome calculations) and give the main results of Eqs. (7), (8), (11).

It turns out that

$$x^3 \approx H^3\kappa^2, \tag{14}$$

where

$$H^3 = \frac{3r_+^3}{4(-\Lambda)(1 - 2\Lambda r_+^2)} \tag{15}$$

and the constants in (12)

$$D = \frac{1}{r_+^2} - 2\Lambda, \tag{16}$$

$$C = -\frac{2}{r_+^3} + \frac{8}{3}\frac{\Lambda}{r_+}. \tag{17}$$

As in the extremal limit $\kappa \to 0$ we must have $f > 0$ in the vicinity of the horizon from the outside, the coefficient $D > 0$. Then, in combination with $H > 0$, this entails that $\Lambda < 0$.