Physics Letters B 689 (2010) 1–7

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Brane-like singularities with no brane

A.V. Yurov

I. Kant Russian State University, Theoretical Physics Department, Al. Nevsky St. 14, Kaliningrad 236041, Russia

ARTICLE INFO

Article history:
Received 1 July 2009
Received in revised form 14 April 2010
Accepted 16 April 2010
Available online 21 April 2010
Editor: S. Dodelson

Keywords:
Singularity with finite value of scale factor
Brane
Scalar field
Tachyon potential

ABSTRACT

We use a method of linearization to study the emergence of the future cosmological singularity characterized by finite value of the cosmological radius. We uncover such singularities that keep Hubble parameter finite while making all higher derivatives of the scale factor (starting out from the $\ddot{a}$) diverge as the cosmological singularity is approached. Since such singularities has been obtained before in the brane world model we name them the "brane-like" singularities. These singularities can occur during the expanding phase in usual Friedmann universe filled with both a self-acting, minimally coupled scalar field and a homogeneous tachyon field. We discover a new type of finite-time, future singularity which is different from type I–IV cosmological singularities in that it has the scale factor, pressure and density finite and nonzero. The generalization of w-singularity is obtained as well.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Starting out from the discovery of the cosmic acceleration [1] there have been constructed many models of the dark energy, including the very unusual ones: the phantom energy, the tachyon cosmologies, the brane worlds etc. Consideration of these models results in some unexpected conclusions about possibility of new cosmological doomsday scenarios: the Big Rip singularity (BRS) [2], the Big Freeze singularity (BFS) [3,4], the Sudden Future singularity (SFS) [5], the Big Boost singularity (BBtS) [6], and the Big Break singularity (BBS) [7,8]. In all these models the evolution ends with the curvature singularity, $|\ddot{a}(t)| \to \infty$, reachable in a finite proper time, say as $t \to t_s$. BRS and BFS both take place in the phantom models but with the different equations of state. In particular, BRS takes place if $w = p/\rho = \mathrm{const} < -1$ whereas BFS occurs for the dark energy in the form of a phantom generalized Chaplygin gas. Models with the SFS, BFS, BBtS and BBS singularities are characterized by a finite value of the cosmological radius but different values of Hubble expansion parameter $H_s = H(t_s)$ and different signs of (divergent) expression $\ddot{a}_s/a$ (cf. also [9]):

$$a_s = \infty, \qquad H_s = +\infty, \qquad \frac{\ddot{a}_s}{a_s} = +\infty, \tag{BRS}$$

$$a_s < \infty, \qquad H_s = +\infty, \qquad \frac{\ddot{a}_s}{a_s} = +\infty, \tag{BFS}$$

$$a_s < \infty, \qquad 0 < H_s < \infty, \qquad \frac{\ddot{a}_s}{a_s} = -\infty, \tag{SFS}$$

$$a_s < \infty, \qquad 0 < H_s < \infty, \qquad \frac{\ddot{a}_s}{a_s} = +\infty, \tag{BBtS}$$

$$a_s < \infty, \qquad H_s = 0, \qquad \frac{\ddot{a}_s}{a_s} = -\infty. \tag{BBS}$$

Remark 1. One of classifications of singularities for the modified gravity was given in [10]; for the classification and discussion concerned with avoiding the singularities in the alternative gravity dark energy models cf. [11]. Another classification of finite-time future singularities (type I–IV singularities) is presented in [12]. According to this classification, the BRS is a singularity of type I, BFS is of type III, SFS and BBtS are type II and BBS — type II with $\rho_s \equiv \rho(t_s) = 0$ (although this is a quite non-trivial special case of a type II singularities). Our classification doesn't contain singularities of IV type (for $t \to t_s$, $a \to a_s$, $\rho \to 0$, $|p| \to 0$ and higher derivatives of H diverge) but as we shall see in Section 6, the classification of Ref. [12] is not exactly complete too: the type IV is the special case of a more general type of singularities.

Remark 2. Another type of "singularity" — so-called w-singularity was obtained in [13]. This "singularity" has a finite scale factor, vanishing energy density and pressure, and the singular behavior manifesting itself only in a time-dependent barotropic index $w(t)$. The w-singularities seem to be most similar to the type IV but are different nonetheless since they do not lead to any divergence of higher order derivatives of H [13].

One surmises that w-singularity is not a correct physical singularity since all the physical values (i.e. density, pressure and higher derivatives of the scale factor or Hubble roots) are finite. Moreover,

E-mail address: artyom_yurov@mail.ru.

0370-2693/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.physletb.2010.04.038

the definition of w-singularity from the [13] is an incomplete one. To show this let us consider the following form of the scale factor

$$a(t) = a_s - A(t_s - t)^m. \tag{1}$$

(1) is the special case of the general form of the scale factor from the [13] (with $B = 0$, $A = a_s$, $C/t_s^n = -A$, $D = 1$, $n = m$). One can show that for $t \to t_s$:

(a) type III singularity if $0 < m < 1$;
(b) type II singularity if $1 < m < 2$;
(c) w-singularity if $m > 2$.

The case $m = 1$ results in a model with the constant barotropic index $w = -1/3$. The case $m = 2$ is the most interesting one because

$$\rho \to 0, \qquad p \to \frac{4A}{3a_s} \neq 0, \qquad |w| \to \infty.$$

and

$$\frac{d^{2n}H}{dt^{2n}} = 0, \qquad \frac{d^{2n+1}H}{dt^{2n+1}} \sim \frac{A^{n+1}}{a_s^{n+1}} < \infty,$$

at $t = t_s$. Thus we have some generalization of w-singularity such that the pressure is non-vanishing and finite at $t = t_s$.

The BRS and BFS have been obtained in the phantom cosmologies (BRS for the phantom perfect fluid with equation of state $p/\rho = w = \text{const} < -1$ and BFS for the phantom Chaplygin models. Throughout the Letter we'll stick to the metric units with $8\pi G/3 = c = 1$). The BBtS is connected to the effect of the conformal anomaly that drives the expansion of the Universe to a maximal value of the Hubble constant, after which the solution becomes complex. The BBS takes place in tachyon models.

Unlike BRS, BFS and BBtS altogether, the BBS and SFS are violating just the dominant energy condition ($\rho \geqslant 0$, $-\rho \leqslant p \leqslant \rho$). It is also possible to obtain some generalization of these singularities. In particular, generalization of the Sudden Future singularities (the so-called Generalized Sudden Future singularities or GSFS) are singularities such that one has the derivative of pressure $p^{(m-2)}$ singularity which accompanies the blow-up of the m-th derivative of the scale factor $a^{(m)}$ [14]. These singularities are possible in theories with higher-order curvature quantum corrections [12] and corresponds to classification in this Letter.

Despite the fact that there has recently been a great inflow of articles, elaborating on the aforementioned singularities, an absolute majority of them has been of a mathematical nature, while the physical reasons for arousal of such singularities still remain less then clear. A remarkable exception is the article [4], which has introduced for the first time a new type of cosmological singularities located on the brane (for discussion about the soft singularities on brane with the quantum corrections cf. [15]). These singularities are characterized by the fact that while the Hubble parameter and scale factor remain finite, all higher derivatives of the scale factor ($\ddot{a}$ etc.) diverge as the cosmological singularity is approached. These singularities may be obtained as the result of embedding of $(3+1)$-dimensional brane in the bulk and this is why these singularities will be henceforth referred to as the "brane-like" singularities. We'll define the "brane-like" singularities in a following fashion: we'll say that *singularity is of a "brane-like" type if at the instance of its occurrence both scale factor and density remain finite and nonzero, while all the higher order derivatives of scale factor (starting with the second order) become altogether singular, i.e.* $a \to a_s$, $\rho \to \rho_s$, $0 < a_s < \infty$, $0 < \rho_s < \infty$, $d^n a_s/dt^n = \infty$ *for* $n > 1$.

Evidently, the class of "brane-like" singularities includes the singularities of type II (with $\rho_s \neq 0$) or SFS and BBtS. Moreover, BBS will also be of this type whenever we are talking about the models with the constant positive curvature, since at the singularity point $\rho_s = 1/a_s^2$.

The physical nature of "brane-like" singularities emergence is quite clear: in the simple case with Z_2 reflection symmetry and the identical cosmological constants on the two sides of the brane, the dynamical equation contains few additional terms. One of them is the square root of the sum of contributions of density (on the brane), tension, cosmological constant and the "dark radiation" (the last one arises due to the projection of the bulk gravitational degrees of freedom onto the brane [4]). This sum is not positively defined and might become negative during the cosmological evolution. Thus, the solution of the cosmological equations can't be continued beyond the point where this sum turns to zero and what we end up with at this point is nothing but a "brane-like" singularity. Since the existence of such singularities is natural in the brane physics, it won't be against the logic to assume that the appearance of "brane-like" singularities in usual Friedmann cosmology (SFS or BBtS) might be an evidence of validity of the brane hypothesis. Therefore it is interesting to consider "brane-like" singularities without a brane (i.e. in FLRW cosmology) to establish the particular form of potential and the equation of state that will result in such singularities during cosmological dynamics. Such potential and the equation of state may altogether be useful for answering the big cosmological question: Don't we really live on the brane?

Furthermore, such singularities may actually result in very unusual models. In fact, let's consider the universe which contains a "brane-like" singularity. If the universe is filled with a scalar field while the Hubble parameter $H(t_s) = H_s$ and the scale factor $a(t_s) = a_s$ are finite at the singular point ($H_s < \infty$, $a_s < \infty$) then the value of the scalar field $\phi(t_s) = \phi_s$ might be finite as well. On the other hand, quantum corrections of higher order (in N-loops approximation) depend on the higher derivatives. If higher derivatives of scale factor diverge then this will also be the case for the scalar field. So one can expect that since all higher derivatives of scale factor and field alike diverge as the cosmological singularity is approached, then the quantum effects will be dominating for $t \to t_s$. This will be the case in spite of the fact that both density ρ_s and scale factor a_s will be finite and that ρ_s might be small and a_s — very large.

It may seem that quantum corrections will be dominating because the pressure $|p| \to \infty$ as the cosmological singularity is approached. This is not the case for the singularities of the IV type being "brane-like" by definition. Moreover, in Section 6 we'll construct the singularities of even more general type that will violate the classifications of [12] since ρ_s and p_s will be finite and nonzero and all higher derivatives will diverge.

In this Letter one constructs "brane-like" singularities in Friedmann–Lemaitre–Robertson–Walker universe filled with the usual self-acting, minimally coupled scalar field or homogeneous tachyon field. In this cosmology we'll also construct the singularities (with finite scale factor) where that Hubble variable vanishes and all higher derivatives of the scale factor diverge as the cosmological singularity is approached. That type of singularities is the generalization of the Big Break singularities and there will also be those of the "brane-like" type for the case of a constant positive curvature. We will calculate both self-acting potential $V(\phi)$ and tachyon potential $V(T)$ that result in appearance of such singularities. Additionally, we'll present the equation of state for such models. Moreover, the classification of singularities from Ref. [12] will be complemented.

This Letter contains a discussion of a simple but useful method which allows one to construct exact solutions of the cosmological Friedmann equations filled with both self-acting, minimally cou-

Download English Version:

https://daneshyari.com/en/article/10723392

Download Persian Version:

https://daneshyari.com/article/10723392

Daneshyari.com