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The asymptotic behavior of Casimir force in the presence of compactified universal extra dimensions

Hongbo Cheng

Department of Physics, East China University of Science and Technology, Shanghai 200237, China

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Abstract

The Casimir effect for parallel plates in the presence of compactified universal extra dimensions within the frame of Kaluza–Klein theory is analyzed. Having regularized and discussed the expressions of Casimir force in the limit, we show that the nature of Casimir force is repulsive if the distance between the plates is large enough and the higher-dimensional spacetime is, the greater the value of repulsive Casimir force between plates is. The repulsive nature of the force is not consistent with the experimental phenomena.

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Unifying the interactions in nature needs a powerful ingredient like the model of higher-dimensional spacetime. Nearly 80 years ago the idea that our universe has more than four dimensions was put forward by Kaluza and Klein [1,2]. In this theory named Kaluza–Klein theory, one extra dimension in our Universe was introduced to be compactified in order to unify gravity and classical electrodynamics. Recently the quantum gravity such as string theories or brane-world scenario is developed to reconcile the quantum mechanics and gravity with the help of introducing seven extra spatial dimensions. In Randall–Sundrum model the matter fields may be localized on a four-dimensional brane considered as our real universe, and only gravitons can propagate in the extra space transverse to the brane [3,4]. In some approaches larger extra dimensions were also invoked for providing a breakthrough of hierarchy problem [5–7]. The order of the compactification scale of the extra dimensions has not been confirmed and are also of considerable interest recently. In a word, studies of higher-dimensional spacetime have therefore been pursed vigorously and extensively and more achievements have been made.

The Casimir effect as a fundamental aspect of quantum field theory in confined geometries and the physical manifestation of zero-point energy has received great attention and has been extensively studied in a wide variety of topics [8–18]. The topics include the influence from the effect on the stability of radion in the Randall–Sundrum model, the cosmological aspects like the cosmological constant and the primordial cosmic inflation [19,20]. The effect was also explored in the context of string theory [21–24]. The precision of the measurement has been greatly improved practically [25–28], leading the Casimir effect to be a remarkable observable and trustworthy consequence of the existence of quantum fluctuations. The experimental results clearly show that the attractive Casimir force between the parallel plates vanishes when the plates move apart from each other to the very distant place. In particular it must be pointed out that no repulsive force appears. Therefore the Casimir effect can become a powerful tool for the study and development of a large class of topics on the model of Universe with more than four dimensions.

Exploring the possible existence or size of extra dimensions by means of Casimir effect attracts more attentions of the physical community. The electromagnetic Casimir effect for parallel plates in higher-dimensional spacetime without compactified universal extra dimensions has been studied and some important results were obtained [29,30]. In that case there are divergences in the Casimir energy at the boundaries unless a careful subtraction was performed. Here we focus on the topic

E-mail address: hbcheng@public4.sta.net.cn (H. Cheng).

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in the frame of Kaluza–Klein theory. In this approach research on the Casimir effect in five-dimensional spacetimes is just the first step of generalization to investigate the higher-dimensional spacetimes. Having examined the Casimir effect for the rectangular cavity in the presence of a compactified universal extra dimension, we show analytically that the extra-dimension corrections to the standard Casimir effect are very manifest [31]. The Casimir effect for parallel plates in the spacetime with one extra compactified dimension was discussed. Only when the plates gap is very small, the size of the additional dimension satisfying $L \leqslant 10$ nm was obtained by comparison to experimental data [32]. We also scrutinized the same problem and show rigorously that there must appear repulsive Casimir force between the parallel plates within the experimental reach when the plates distance is large enough in the spacetime with one compactified additional dimension [33]. Therefore the results obtained from the Kaluza–Klein theory including only one compactified spatial extra dimension are not consistent with the experimental results mentioned above, which means that the model that the spacetime with only one extra dimension cannot be realistic.

As mentioned above a lot of models such as the string theories motivate the models with more than five dimensions, suggesting it is necessary to continue exploring the Casimir effect in the presence of more compactified universal extra dimensions in detail in order to know whether the models of spacetime with more than one additional dimensions are realistic. This problem, to our knowledge, has not been discussed. For simplicity and comparison to the measurement the system consisting of two parallel plates is always chosen. The purpose of this Letter is to reexamine the Casimir effect for parallel plates in the universe with d compactified spatial dimensions carefully. We regularize the total energy to obtain the Casimir energy, and then Casimir force. In particular we focus on the asymptotic behaviour of the Casimir force between plates for their large enough gap and the dependence of dimensionality of the spacetime in order to compare our results with the measuring evidence listed above directly. Finally the conclusions are emphasized.

In the Kaluza–Klein theory we start to consider the scalar field in the system consisting of two parallel plates in the spacetime with d extra compactified dimensions. Along the extra dimensions the wave vectors of the field have the form $k_i = \frac{n_i}{L}$, $i = 1, 2, \ldots, d$, respectively, n_i an integer. Here we choose that the extra dimensions possess the same radius as L. At the plates the fields satisfy the Dirichlet condition, leading the wave vector in the directions restricted by the plates to be $k_n = \frac{n\pi}{R}$, n a positive integer and R the separation of the plates. Under these conditions, the zero-point fluctuations of the fields can give rise to observable Casimir forces.

In the case of d additional compactified dimensions we find the frequency of the vacuum fluctuations to be

$$\omega_{\{n_i\}n} = \sqrt{k^2 + \frac{n^2\pi^2}{R^2} + \sum_{i=1}^{d} \frac{n_i^2}{L^2}}, \tag{1}$$

where

$$k^2 = k_1^2 + k_2^2, \tag{2}$$

k_1 and k_2 are the wave vectors in directions of the unbound space coordinates parallel to the plates surface. Here $\{n_i\}$ represents a short notation of $n_1, n_2, \ldots, n_d$, n_i a nonnegative integer. Following Refs. [9–16], therefore the total energy density of the fields in the interior of system reads,

$$\begin{aligned}\varepsilon &= \int \frac{d^2k}{(2\pi)^2} \sum_{n=1}^{\infty} \sum_{\{n_i\}=0}^{\infty} \frac{1}{2}\omega_{\{n_i\}n} \\ &= \frac{\pi}{2} \frac{\Gamma(-\frac{3}{2})}{\Gamma(-\frac{1}{2})} \sum_{l=0}^{d-1} \binom{d}{l} E_{d-l+1}\left(\frac{\pi^2}{R^2}, \frac{1}{L^2}, \frac{1}{L^2}, \ldots, \frac{1}{L^2}; -\frac{3}{2}\right) \\ &\quad + \frac{\pi^3}{2R^3} \frac{\Gamma(-\frac{3}{2})\zeta(-3)}{\Gamma(-\frac{1}{2})}\end{aligned} \tag{3}$$

in terms of the Epstein zeta function $E_p(a_1, a_2, \ldots, a_p; s)$ defined as

$$E_p(a_1, a_2, \ldots, a_p; s) = \sum_{\{n_j\}=1}^{\infty} \left(\sum_{j=1}^{p} a_j n_j^2\right)^{-s} \tag{4}$$

here $\{n_j\}$ stands for a short notation of $n_1, n_2, \ldots, n_p$, n_j a positive integer. We regularize Eq. (3) by means of the following result,

$$\begin{aligned}&\Gamma\left(-\frac{3}{2}\right) E_{d-l+1}\left(\frac{\pi^2}{R^2}, \frac{1}{L^2}, \frac{1}{L^2}, \ldots, \frac{1}{L^2}; -\frac{3}{2}\right) \\ &\quad = -\frac{1}{2}\Gamma\left(-\frac{3}{2}\right) E_{d-l}\left(1, 1, \ldots, 1; -\frac{3}{2}\right) \frac{1}{L^3} \\ &\qquad + \frac{1}{L^3} \sum_{k=0}^{\infty} \frac{16^{-k}}{k!} \left(\frac{R}{L}\right)^{-k-\frac{3}{2}} \prod_{j=1}^{k} \left[16 - (2j-1)^2\right] \\ &\qquad \times \sum_{n_1, n_2, \ldots, n_{d-l+1}=1}^{\infty} n_1^{-k-\frac{5}{2}} \left(n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2\right)^{-\frac{2k-3}{4}} \\ &\qquad \times \exp\left[-\frac{2R}{L} n_1 \left(n_2^2 + n_3^2 + \cdots + n_{d-l+1}^2\right)^{\frac{1}{2}}\right] \\ &\qquad + \frac{\Gamma(-2)}{2\sqrt{\pi}} E_{d-l}(1, 1, \ldots, 1; -2) \frac{R}{L^4}\end{aligned} \tag{5}$$

to obtain the Casimir energy density of parallel plates in the spacetime with d extra compactified spatial dimensions. It is certainly fundamental to investigate the Casimir force for the same system in the same background in order to compare our results with the experimental phenomenon. The Casimir force is given by the derivative of the Casimir energy with respect to the plate distance. Here we focus on the property as the plate distance R approaches to the infinity, then the expression for the Casimir force in the limiting case is defined as

$$f = -\lim_{\mu \longrightarrow \infty} \frac{\partial \varepsilon}{\partial R}, \tag{6}$$

where

$$\mu = \frac{R}{L}. \tag{7}$$