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Pairing in fermion systems with unequal masses: Nonperturbative renormalisation group approach

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Abstract

The application of the nonperturbative renormalisation group approach to a system with two fermion species is studied. Assuming a simple ansatz for the effective action with effective bosons, describing pairing effects we derive a set of approximate flow equations for the effective coupling including boson and fermionic fluctuations. The case of two fermions with different masses but coinciding Fermi surfaces is considered. The phase transition to a phase with broken symmetry is found at a critical value of the running scale. The large mass difference is found to disfavour the formation of pairs. The mean-field results are recovered if the effects of boson loops are omitted. While the boson fluctuation effects were found to be negligible for large values of $p_F a$ they become increasingly important with decreasing $p_F a$ thus making the mean field description less accurate.

Keywords: Nonperturbative renormalisation group; EFT; Broken phase; Superfluidity

The properties of asymmetric many fermion systems have recently attracted much attention (see, for example Ref. [1] and references therein) driven by the substantial advance in experimental studies of trapped fermionic atoms. This asymmetry can be provided by unequal masses, different densities and/or chemical potentials. Understanding the pairing mechanism in such settings would be of immense value for different many fermion systems from atomic physics to strongly interacting quark matter. The important theoretical issue to be resolved here is the nature of the ground state. Several competing states have been proposed so far. These include: LOFF [2] phase, breached-pair (BP) superfluidity [3] (or Sarma phase) and mixed phase [4]. Establishing the true ground state is still an open question. It was shown, for example, that LOFF and mixed phases are more stable then the Sarma phase in the systems of fermions with the mismatched Fermi surfaces and with both equal and different masses [1,4,5]. All these studies, however, have been performed within the mean field approximation (MFA). In spite of the fact that in many cases MFA is quite reliable it is important to understand better the limits of applicability of MFA in the context of the fermion systems with a certain type of asymmetry (masses and/or densities) and work out the physical regimes where the MFA is too crude or even inadequate. The convenient way to estimate the corrections to MFA is provided by the nonperturbative renormalisation group (NRG) approach [6] which was successfully applied to the standard pairing problem with one type of fermions [7–10]. The main element of NRG is the effective average action Γ_k which is a generalisation of the standard effective action Γ, the generating functional of the 1PI Green functions. The only difference between them is that Γ_k includes only quantum fluctuations with momenta larger then the infrared scale k. The evolution of the system as the function of the scale k is described by the nonperturbative flow equations. When $k \to 0$ all fluctuations are included and full effective action is recovered. Similarly, at starting scale $k = K$ no fluctuations are included so $\Gamma_{k=K}$ can be associated with the classical action S therefore Γ_k provides an interpolation between the classical and full quantum effective actions.

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The dependence of Γ_k from the infrared scale k is given by the nonperturbative renormalisation group equation (NRGE)

$$\partial_k \Gamma = -\frac{i}{2}\,\mathrm{Tr}\big[(\partial_k R)\big(\Gamma^{(2)} - R\big)^{-1}\big]. \tag{1}$$

Here $\Gamma^{(2)}$ is the second functional derivative of the effective action taken with respect to all types of field included in the action and $R(q,k)$ is a regulator which should suppress the contributions of states with momenta less than or of the order of running scale k. To recover the full effective action we require $R(q,k)$ to vanish as $k \to 0$ whereas for $q \ll k$ the regulator behaves as $R(q,k) \simeq k^2$. The above written equation is, in general, the functional equation. For a practical applications it needs to be converted to the system of partial or ordinary differential equations so that approximations and truncations are required.

We consider a nonrelativistic many-body system at zero temperature with two types of the fermion species a and b interacting through a short-range attractive interaction and introduce a boson field ϕ describing the pair of interacting fermions. The ansatz for Γ takes the form

$$\begin{aligned}
&\Gamma\big[\psi,\psi^\dagger,\phi,\phi^\dagger,\mu,k\big] \\
&\quad= \int d^4x \Bigg[\phi^\dagger(x)\bigg(Z_\phi(i\partial_t + \mu_a + \mu_b) + \frac{Z_m}{2m}\nabla^2\bigg)\phi(x) \\
&\qquad - U\big(\phi,\phi^\dagger\big) + \sum_{i=a}^{b}\psi_i^\dagger\bigg(Z_{\psi,i}(i\partial_t + \mu_i) + \frac{Z_{M,i}}{2M_i}\nabla^2\bigg)\psi_i \\
&\qquad - Z_g\bigg(\frac{i}{2}\psi_b^{\mathrm{T}}\sigma_2\psi_a\phi^\dagger - \frac{i}{2}\psi_a^\dagger\sigma_2\psi_b^{\dagger\mathrm{T}}\phi\bigg)\Bigg].
\end{aligned} \tag{2}$$

Here M_i is the mass of the fermion in vacuum and the factor $1/2m$ with $m = M_a + M_b$ in the boson kinetic term is chosen simply to make Z_m dimensionless. The coupling Z_g, the wave-function renormalisations factors $Z_{\phi,\psi}$ and the kinetic-mass renormalisations factors $Z_{m,M}$ all run with k, the scale of the regulator. Having in mind the future applications to the crossover from BCS to BEC (where chemical potential becomes negative) we also let the chemical potentials μ_a and μ_b run, thus keeping the corresponding densities (and Fermi momenta $p_{F,i}$) constant. The bosons are, in principle, coupled to the chemical potentials via a quadratic term in ϕ, but this can be absorbed into the potential by defining $\bar{U} = U - (\mu_1 + \mu_2)Z_\phi\phi^\dagger\phi$. The evolution equations include running of chemical potentials, effective potential and all couplings $(Z_\phi, Z_m, Z_{M,i}, Z_{\psi,i}, Z_g)$. However, in this Letter we allow to run only Z_ϕ, parameters in the effective potential ($u's$ and ρ_0) and chemical potentials since this is the minimal set needed to include the effective boson dynamics.

We expand the effective potential about its minimum, $\phi^\dagger\phi = \rho_0$, so that the coefficients u_i are defined at $\rho = \rho_0$,

$$\begin{aligned}
\bar{U}(\rho) &= u_0 + u_1(\rho - \rho_0) + \frac{1}{2}u_2(\rho - \rho_0)^2 \\
&\quad + \frac{1}{6}u_3(\rho - \rho_0)^3 + \cdots,
\end{aligned} \tag{3}$$

where we have introduced $\rho = \phi^\dagger\phi$. Similar expansion can be written for the renormalisation factors. The coefficients of the expansion run with the scale. The phase of the system is determined by the coefficient u_1. We start evolution at high scale where the system is in the symmetric phase so that $u_1 > 0$. When the running scale becomes comparable with the pairing scale (close to average Fermi momentum) the system undergoes the phase transition to the phase with broken symmetry, energy gap etc. The point of the transition corresponds to the scale where $u_1 = 0$. The bosonic excitations in the gapped phase are gapless Goldstone bosons. Note, that in this phase the minimum of the potential will also run with the scale k so that the value $\rho_0(k \to 0)$ determines the physical gap.

The evolution equation takes the following general form

$$\begin{aligned}
\partial_k \Gamma &= -\frac{i}{2}\,\mathrm{Tr}\big[(\partial_k R_B)\big(\Gamma^{(2)}_{BB} - R_B\big)^{-1}\big] \\
&\quad + \frac{i}{2}\,\mathrm{Tr}\big[(\partial_k R_F)\big(\Gamma^{(2)}_{FF} - R_F\big)^{-1}\big].
\end{aligned} \tag{4}$$

Here $\Gamma^{(2)}_{BB(FF)}$ is the matrix of the second functional derivatives of the effective action taken with respect to boson (fermion) fields included in the action and R_B (R_F) is the boson (fermion) regulator which should suppress the contributions of states with momenta less than or of the order of running scale k. The boson regulator has the structure

$$\mathbf{R}_B = R_B\,\mathrm{diag}(1,1). \tag{5}$$

The fermion regulator for both types of fermions has the structure

$$\mathbf{R}_{F,i} = \mathrm{sgn}\big(\epsilon_i(q) - \mu_i\big)R_{F,i}(q,\mu_i,k)\,\mathrm{diag}(1,-1). \tag{6}$$

Note that this regulator is positive for particle states above the Fermi surface and negative for the hole states below the Fermi surface.

Calculating the second functional derivatives, taking the matrix trace and carrying out the pole integration in the loop integrals we get the evolution equation for U at constant chemical potentials

$$\begin{aligned}
\partial_k \bar{U} &= -\frac{1}{\mathcal{V}_4}\partial_k\Gamma \\
&= -\frac{1}{2}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{E_{F,S}}{\sqrt{E_{F,S}^2 + \Delta^2}} \\
&\quad\times\big[\mathrm{sgn}(q - p_{\mu,a})\partial_k R_{F,a} + \mathrm{sgn}(q - p_{\mu,b})\partial_k R_{F,b}\big] \\
&\quad + \frac{1}{2Z_\phi}\int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\frac{E_B}{\sqrt{E_B^2 - V_B^2}}\partial_k R_B.
\end{aligned} \tag{7}$$

Here

$$\begin{aligned}
&E_S = (E_{F,a} + E_{F,b})/2, \\
&E_A = (E_{F,a} - E_{F,b})/2,
\end{aligned} \tag{8}$$

and

$$\begin{aligned}
&E_B(q,k) = \frac{Z_m}{2m}q^2 + u_1 + u_2\big(2\phi^\dagger\phi - \rho_0\big) + R_B(q,k), \\
&V_B = u_2\phi^\dagger\phi,
\end{aligned} \tag{9}$$