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Some considerations about NS5 and LST Hawking radiation

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ABSTRACT

We have studied the Hawking radiation corresponding to the NS5 and Little String Theory (LST) black hole models using two semi-classical methods: the complex path method and a gravitational anomaly. After summarizing some known concepts about the thermodynamics of these theories, we have computed the emission rates for the two black hole models. The temperature calculated from, e.g. the well-known surface gravity expression, is shown to be identical to that obtained from both the computation of the gravitational anomaly and the complex path method. Moreover, the two semi-classical methods show that NS5 exhibits non-thermal behavior that contrasts with the thermal behavior of LST. We remark that energy conservation is the key factor leading to a non-thermal profile for NS5. In contrast, LST keeps a thermal profile even when energy conservation is considered because temperature in this model does not depend on energy.

1. Introduction

Since the pioneering proposal of Hawking that black holes can radiate [1], much work has been done in order to obtain a complete theory of quantum gravity. When Hawking announced his amazing results, a new powerful paradox emerged. The information loss paradox with the apparent violation of unitarity principle has consequences on well-established quantum mechanics. A recent effort in order to solve this paradox has been done studying different semi-classical approaches such as the tunneling method proposed by Parikh and Wilczek [2,3], the complex path analysis [4–6] or the cancellation of gravitational anomalies [7–9].

We have studied the Hawking radiation for NS5 and LST stringy black holes using different semi-classical methods obtaining equal results for each method. For good reviews on LST and NS5, we address the readers to [10–15]. We have verified that the NS5 model shows a non-thermal emission whereas LST shows a thermal emission. This last conclusion matches with the Hagedorn properties of LST, namely the temperature of LST corresponds to the Hagedorn temperature.

The Letter is organized as follows: in Section 2, we briefly summarize the LST theory with some properties and thermodynamics. In Section 3, we reduce the ten-dimensional metric of LST to two-dimensional one. All the physics will be analyzed within the propagation of massless particles in the r–t sector of the metric. In Section 4, we study two different semi-classical methods in order to compute the emission rate for NS5 and LST. Complex path method and anomalies yields the same results as the tunneling method, analyzed in [16], for the temperature and the emission rate. It is worth to mentioning that in the classical computation of the Bogoliubov coefficients all the results for emission rates shows thermal profiles due to the lack of energy conservation. This fact had driven Hawking to state that all the information that falls into the black hole is lost for ever, establishing in this way the information loss paradox. Nevertheless, one hopes to overcome this weird conclusion using semi-classical methods.

2. A glance at LST thermodynamics

Little string theory is a non-gravitational six-dimensional and non-local field theory [10–12,17,18], believed to be dual to a string theory background, defined as the decoupled theory on a stack of N NS5-branes. In the limit of a vanishing asymptotic value for the string coupling $g_s \to 0$, keeping the string length l_s fixed while the energy above extremality is fixed, i.e. $\frac{E}{m_s} = \text{fixed}$, the processes in which the modes that live on the branes are emitted into the bulk as closed strings are suppressed. The theory becomes free in the bulk, but strongly interacting on the brane. In this limit, the theory reduces to Little String Theory or more precisely to $(2,0)$ LST for type IIA NS5-branes and to $(1,1)$ LST for type IIB NS5-branes [15].

The throat geometry corresponding to N coincident non-extremal NS5-branes in the string frame [19] is

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$$ds^2 = -f(r)\,dt^2 + \frac{A(r)}{f(r)}\,dr^2 + A(r)r^2\,d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2, \tag{2.1}$$

where dx_j^2 corresponds to flat spatial directions along the 5-branes, $d\Omega_3^2$ corresponds to 3-sphere of the transverse geometry and the dilaton field is defined as $e^{2\Phi} = g_s^2 A(r)$. The metric functions are defined as

$$f(r) = 1 - \frac{r_0^2}{r^2}, \qquad A(r) = \chi + \frac{N}{m_s^2 r^2}, \tag{2.2}$$

r_0 is the non-extremality parameter, so the extremal configuration is obtained by the limit $r_0 \to 0$ and the location of the event horizon corresponds to $r = r_0$. We define the parameter χ which takes the values 1 for NS5 model and 0 for LST, these are only the values for which exist a supergravity solution. In addition to the previous fields there is an *NS–NS* $H_{(3)}$ form along the S^3, $H_{(3)} = 2N\epsilon_3$. According to the holographic principle the high spectrum of this dual string theory should be approximated by certain black hole in the background (2.1). The geometry transverse to the 5-branes is a long tube which opens up into the asymptotic flat space with the horizon at the other end, in the limit $r \to r_0$ appears the semi-infinite throat parametrized by (t, r) coordinates, in this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat. The string propagation in this geometry should correspond to an exact conformal field theory [20]. The boundary of the near horizon geometry is $R^{5,1} \times R \times S^3$. The geometry (2.1) is regular as long as $r_0 \neq 0$. In order to construct the thermal states of the black holes we write the corresponding metric in Rindler coordinates as

$$ds^2 = -\kappa^2\rho^2\,dt^2 + d\rho^2 + A(r)r^2\,d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2, \tag{2.3}$$

where we have introduced the radial Rindler coordinate ρ (proper length), also we see that the quantity $\kappa = \frac{f(r_0)'}{2\sqrt{A(r_0)}}$ coincides with the surface gravity of the NS5 and LST black holes. Then performing a Wick rotation $t_E = it$, the Euclidean time coordinate has to be periodic with period 2π in order to avoid a conical singularity. Thus we identify the period of the Euclidean time as in [21]

$$\beta = \frac{2\pi}{\kappa} = \frac{2\pi\sqrt{N + \chi r_0^2}}{m_s}. \tag{2.4}$$

The temperature obtained, $T = \beta^{-1}$, does not depend on which frame we are using, the string frame and Einstein frame are related by a local rescaling that does not affect the result.

Regarding LST model, we notice that the temperature is independent of the black hole radius and therefore of the black hole mass. In this way we could identify this temperature with the Hagedorn temperature of the superstring theory. One can compute the energy density for the LST background in ten dimensions, $e \equiv \frac{E}{V_5}$ and the entropy density, $s \equiv \frac{S}{V_5}$, where V_5 is the volume of the flat 5-branes space and S is the standard Bekenstein–Hawking entropy calculated from the area of the event horizon of the black hole, $S = \frac{\text{Area}}{4G_{10}}$. Either in Einstein frame metric [21,22] or in string frame metric [14,19] it is satisfied the usual thermodynamic relation $S = \beta E$. This relation implies that the free energy of the system $F = E - TS$ vanishes. At very high energies the equation of state is of the Hagedorn form which leads to an exponentially growing density of states [13]: $\rho(E) = e^{S(E)} \sim e^{\beta E}$.

At first sight one could think that a phase transition is present when the system evolves from NS5 to the near horizon limit of NS5, i.e. LST, but we have checked that it is not the case. Computing the Bekenstein–Hawking entropy

$$S = \frac{\text{Area}}{4G_{10}} = \frac{V_5}{2G_{10}}\pi^2\left(N + \chi m_s^2 r_0^2\right)^{3/2}, \tag{2.5}$$

and plotting it versus the temperature we do not detect any critical point (Davies point) [23] that would signal a phase transition. Even working in thermodynamic geometry [24], writing the LST metric like a Ruppeiner metric $ds^2 = -3\sqrt{\frac{\pi G}{\hbar^2 M}}\,dS^2$, we do not detect any divergence in the scalar curvature that would signal a possible phase transition. However calculating the specific heat as $C = T\frac{\partial S}{\partial T}$, we have found that it has a negative value, $-3S$, showing that the theory is unstable. In the work [13], the authors show that loop/string corrections to the Hagedorn density of states of LST would be of the form $\rho(E) \sim E^{\alpha} e^{\beta E}(1 + O(\frac{1}{E}))$ and the temperature–energy relation becomes $\beta = \frac{\partial \log\rho}{\partial E} = \beta_0 + \frac{\alpha}{E} + O(\frac{1}{E^2})$. They found that since α is negative the high energy thermodynamics corresponding to near-extremal 5-branes is unstable as well as the temperature is above the Hagedorn temperature and the specific heat is negative. This instability would be associated to the presence of a negative mode (tachyon) in string theory, the high temperature phase of the theory yields the condensation of this mode. The authors are lead again to the conclusion that the Hagedorn temperature is reached at a finite energy being associated with a phase transition.

Next we would like to address the question whether an observer in a moving frame observes a temperature above the Hagedorn temperature. We know that in the near horizon limit of NS5, i.e. LST, the system reaches the maximum temperature, namely the Hagedorn temperature. One could think that a boosted observer may observes a temperature bigger than the Hagedorn one, for this reason we want to verify the validity of this statement. We have evaluated the simplest case, a scalar particle-like observer which moves on an NS5-brane with constant velocity at fixed distance r from the horizon of the LST black hole. We consider the orbit for which $y_1 = vt$. Relating the time coordinate t with the proper time τ through $d\tau^2 = -g_{\mu\nu}\,dx^{\mu}\,dx^{\nu}$, one obtains

$$\frac{d\tau}{dt} = \sqrt{f(r) - v^2}. \tag{2.6}$$

The velocity is bounded by the local velocity of light thus we have to impose the constraint $v^2 \leqslant f(r)$. This relation brings us to a new coordinate of the horizon position, where causality is lost, seen by the moving particle, $r = \frac{r_0}{\sqrt{1-v^2}}$. Furthermore the Killing vector relevant for the process is $\zeta = -\partial_t + v\partial_{y_1}$. Therefore evaluating the surface gravity at this new coordinate r, we obtain the local temperature for the moving scalar particle

$$T' = \frac{(1-v^2)m_s}{2\pi\sqrt{N + \chi\frac{r_0^2}{(1-v^2)}}}, \tag{2.7}$$

where we have worked out in natural units, $c = 1$ and $v < 1$. We notice two important features. First of all, we see that in the $v \to 0$ limit we recover the result (2.4). Secondly, comparing the temperature for the particle-like observer (2.7) with the temperature defined by (2.4) for an asymptotic static observer, we see that the former is lower than the second one. We conclude that the Hawking temperature of LST is a maximum bound and corresponds to the Hagedorn temperature. Unfortunately, we are not able to perform the same analysis for an accelerating particle-like observer. The main problem is that the path which the particle follows is not generated by a Killing vector field, this fact prevent us from using the surface gravity method in order to calculate the temperature.