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Casimir effect in spacetime with extra dimensions – from Kaluza–Klein to Randall–Sundrum models

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ABSTRACT

In this Letter, we derive the finite temperature Casimir force acting on a pair of parallel plates due to a massless scalar field propagating in the bulk of a higher dimensional brane model. In contrast to previous works which used approximations for the effective masses in deriving the Casimir force, the formulas of the Casimir force we derive are exact formulas. Our results disprove the speculations that existence of the warped extra dimension can change the sign of the Casimir force, be it at zero or any finite temperature.

1. Introduction

Since the advent of string theory, theories of spacetime with extra dimensions become prevalent in physics. The idea of extra dimensional spacetime can be dated back to the work of Kaluza and Klein [1,2], who tried to propose a theory that can unify classical electrodynamics and gravity. Recently, intensive investigations on the Casimir effect in spacetime with extra dimensions are undergoing. In the context of string theory, Casimir effect was studied in [3–6]. The possible roles played by Casimir energy as dark energy or cosmological constant was discussed in [7–15]. The use of Casimir effect in stabilizing extra dimensions were considered in [16–25]. In the braneworld scenario, Casimir effect was also considered in [26–34]. The influence of the extra dimensions on the Casimir force acting on a pair of parallel plates in macroscopic $(3+1)$-dimensional spacetime was studied in [13,35–50]. In the pioneering work of Casimir [51], it was shown that the Casimir force acting on a pair of parallel perfectly conducting plates in $(3+1)$-dimensional spacetime is attractive. It was confirmed later in the work of Mehra [52] and Brown and Maclay [53] that the thermal correction would not change the sign of the Casimir force. The recent works [13,35–50] explored the possible influence of the extra dimensions to the magnitude and sign of the Casimir force. In [13,35–37], the Casimir effect on a pair of parallel plates in spacetime with one extra dimension compactified to a circle was considered. Generalizations to extra dimensional space with more dimensions and more complicated geometries were considered in [38–41]. Further generalizations to finite temperature Casimir effect were studied in [42–44]. In the works [35–44], the spacetimes considered are the generalized Kaluza–Klein (KK) models of the form $M^{3+1}\times N^n$ with metric

$$ds^2 = g_{\mu\nu}^{KK}\,dx^\mu\,dx^\nu = \eta_{\alpha\beta}\,dx^\alpha\,dx^\beta - G_{ab}\,dx^a\,dx^b, \quad 0\leqslant \mu,\nu\leqslant n+3,\ 0\leqslant\alpha,\beta\leqslant 3,\ 4\leqslant a,b\leqslant n+3, \tag{1}$$

where $\eta_{\alpha\beta}=\mathrm{diag}(1,-1,-1,-1)$ is the usual $(3+1)$-D metric on the Minkowski spacetime M^{3+1} and $ds_N^2 = G_{ab}\,dx^a\,dx^b$ is a Riemannian metric on the n-dimensional compact internal space N^n. In this model, the metric is factorizable. Hence the geometrical structures of

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the macroscopic manifold M^{3+1} and the internal manifold N^n are independent. It has been concluded that the Casimir force acting on a pair of parallel plates due to a scalar field with homogeneous boundary conditions where Dirichlet conditions are imposed on both plates (*DD* conditions) or Neumann conditions are imposed on both plates (*NN* conditions) is always attractive, at either zero or any finite temperature. On the other hand, for mixed boundary conditions where one of the plates assumes Dirichlet boundary condition and the other one assumes Neumann boundary condition (*DN* conditions), the Casimir force is always repulsive.

In [45–48], the spacetime considered is the Randall–Sundrum (RS) brane model. This model was proposed in [54,55] to solve the hierarchy problem between the Planck and the electroweak scale. In this model, the underlying spacetime is a five-dimensional Anti-de Sitter space (AdS_5) with background metric

$$ds^2 = g^{RS}_{\mu\nu}\,dx^\mu\,dx^\nu = e^{-2\kappa|y|}\eta_{\alpha\beta}\,dx^\alpha\,dx^\beta - dy^2, \quad 0 \leqslant \mu, \nu \leqslant 4,\ 0 \leqslant \alpha, \beta \leqslant 3. \tag{2}$$

This metric is non-factorizable. The extra dimension with coordinate y is compactified on the orbifold $S^1/\mathbb{Z}_2$. The metric of the underlying Minkowski spacetime depends on the extra dimension through the warp factor $e^{-2\kappa|y|}$, where κ determines the degree of curvature of the AdS_5 space. There are two types of RS brane models, denoted by RSI and RSII, respectively. In RSI, there are two 3-branes with equal and opposite tensions, one invisible and one visible, localized at $y = 0$ and $y = \pi R_0$, respectively, where R_0 is the compactification radius of the extra dimension. The $\mathbb{Z}_2$-symmetry is realized by $y \leftrightarrow -y$, $\pi R_0 + y \leftrightarrow \pi R_0 - y$. The standard model fields are localized on the visible brane. RSII can be considered as a limiting case of RSI where $R_0 \to \infty$, i.e., one brane is located at infinity. In relation to Casimir effect on parallel plates, RS model was generalized to $(3+n)$-branes with n-compact dimensions compactified to an n-torus embedded in a $(5+n)$-dimensional spacetime with background metric

$$ds^2 = e^{-2\kappa|y|}\left(\eta_{\alpha\beta}\,dx^\alpha\,dx^\beta - \sum_{i=1}^{n} R_i^2\,d\theta_i^2\right) - dy^2. \tag{3}$$

In [45] and [47], it was concluded that the zero temperature Casimir force acting on a pair of parallel plates in either the $(4+1)$-D RS model (2) or its extension (3) due to a massless scalar field with *DD* boundary conditions is always attractive. The methods used in [45,47] involve approximations to the tower of masses induced by the extra dimension $S^1/\mathbb{Z}_2$, and the attractive nature of the Casimir force is not obvious from its analytical expressions. It is also not clear whether the approximations used in deriving the Casimir force would affect the conclusion about the sign of the Casimir force. Therefore, it is desirable to obtain an exact expression for the Casimir force.

As mentioned in [56], the RS scenario is the simplest case of warped geometries. The higher dimensional warped geometries deserve more attention especially in connection with string theory, which asserts that our spacetime should has eleven dimension. In this Letter, we consider generalized RS model as in [15,31–33,57–59] whose background metric is

$$\begin{gathered}ds^2 = g^{\mathrm{RSKK}}_{\mu\nu}\,dx^\mu\,dx^\nu = e^{-2\kappa|y|}\big(g^{KK}_{\mu\nu}dx^\mu dx^\nu\big) - dy^2 = e^{-2\kappa|y|}\big(\eta_{\alpha\beta}\,dx^\alpha\,dx^\beta - G_{ab}\,dx^a\,dx^b\big) - dy^2,\\ 0 \leqslant \mu, \nu \leqslant n+4,\ 0 \leqslant \alpha, \beta \leqslant 3,\ 4 \leqslant a, b \leqslant n+3.\end{gathered} \tag{4}$$

Compared to the model (3), the internal space now is an arbitrary n-dimensional compact manifold with Riemannian metric $ds_N^2 = G_{ab}\,dx^a\,dx^b$. It can be considered as a KK model (1) embedded in a RS model (2). Therefore we call this model Randall–Sundrum–Kaluza–Klein (RSKK) model. Our concern here is the Casimir force acting on a pair of parallel plates rather than the Casimir force acting on the branes which was considered in [15,31–33]. As in most of the works about Casimir effect on parallel plates in higher dimensional spacetime, we regard the parallel plates as co-dimension one hyperplanes in the spacetime, and the field is assumed to propagate in the bulk. We derive the exact formulas for the finite temperature Casimir force and show that warped extra dimensions cannot change the attractive or repulsive nature of the Casimir force. More precisely, it will be concluded that for *DD* or *NN* boundary conditions, the Casimir force is always attractive; whereas for *DN* boundary conditions, the Casimir force is always repulsive.

The units used are such that $\hbar = c = k_B = 1$.

2. Casimir force on parallel plates in Randall–Sundrum–Kaluza–Klein models

In this section, we derive the Casimir force acting on a pair of parallel plates in the RSKK model with background metric (4) due to a scalar field $\Psi(x, y)$ of mass m with equation of motion

$$\left(\frac{1}{\sqrt{|g^{\mathrm{RSKK}}|}}\sum_{\mu=0}^{n+4}\sum_{\nu=0}^{n+4}\partial_\mu\sqrt{\left|g^{\mathrm{RSKK}}\right|}\big(g^{\mathrm{RSKK}}\big)^{\mu\nu}\partial_\nu + m^2\right)\Psi(x, y) = 0. \tag{5}$$

Using separation of variables,

$$\Psi(x, y) = \varphi(x)\psi(y),$$

the equation of motion (5) for $y \geqslant 0$ is equivalent to the following two equations:

$$e^{(n+2)\kappa y}\frac{d}{dy}\left(e^{-(n+4)\kappa y}\frac{d\psi(y)}{dy}\right) - m^2 e^{-2\kappa y}\psi(y) = -m_{\mathrm{eff}}^2\psi(y), \tag{6}$$

$$\left(\frac{1}{\sqrt{|g^{KK}|}}\sum_{\mu=0}^{n+3}\sum_{\nu=0}^{n+3}\partial_\mu\sqrt{\left|g^{KK}\right|}\big(g^{KK}\big)^{\mu\nu}\partial_\nu + m_{\mathrm{eff}}^2\right)\varphi(x) = 0. \tag{7}$$