Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Physics Letters B 638 (2006) 61–67
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Next-to-next-to-leading order evolution of non-singlet fragmentation functions

A. Mitov [a], S. Moch [a], A. Vogt [b,*]

[a] *Deutsches Elektronensynchrotron DESY, Platanenallee 6, D-15735 Zeuthen, Germany*
[b] *IPPP, Department of Physics, University of Durham, South Road, Durham DH1 3LE, United Kingdom*

Received 19 April 2006; accepted 4 May 2006

Available online 15 May 2006

Editor: N. Glover

Abstract

We have investigated the next-to-next-to-leading order (NNLO) corrections to inclusive hadron production in e^+e^- annihilation and the related parton fragmentation distributions, the 'time-like' counterparts of the 'space-like' deep-inelastic structure functions and parton densities. We have re-derived the corresponding second-order coefficient functions in massless perturbative QCD, which so far had been calculated only by one group. Moreover we present, for the first time, the third-order splitting functions governing the NNLO evolution of flavour non-singlet fragmentation distributions. These results have been obtained by two independent methods relating time-like quantities to calculations performed in deep-inelastic scattering. We briefly illustrate the numerical size of the NNLO corrections, and make a prediction for the difference of the yet unknown time-like and space-like splitting functions at the fourth order in the strong coupling constant.

In this Letter we address the evolution of the parton fragmentation distributions D^h and the corresponding fragmentation functions F_a^h in e^+e^- annihilation, $e^+e^- \to \gamma, Z \to h + X$ where

$$\frac{1}{\sigma_{\rm tot}} \frac{d^2\sigma}{dx\, d\cos\theta} = \frac{3}{8}\left(1+\cos^2\theta\right) F_T^h + \frac{3}{4}\sin^2\theta\, F_L^h + \frac{3}{4}\cos\theta\, F_A^h. \tag{1}$$

Here θ represents the angle (in the center-of-mass frame) between the incoming electron beam and the hadron h observed with four-momentum p, and the scaling variable reads $x = 2pq/Q^2$ where q with $q^2 \equiv Q^2 > 0$ is the momentum of the virtual gauge boson. The transverse (T), longitudinal (L) and asymmetric (A) fragmentation functions in Eq. (1) have been measured especially at LEP, see Ref. [1] for a general overview. Disregarding corrections suppressed by inverse powers of Q^2, these observables are related to the universal fragmentation distributions D^h by

$$F_a^h\left(x, Q^2\right) = \sum_{\rm f=q,\bar q,g} \int_x^1 \frac{dz}{z} c_{a,\rm f}\left(z, \alpha_{\rm s}\left(Q^2\right)\right) D_{\rm f}^h\left(\frac{x}{z}, Q^2\right). \tag{2}$$

The coefficient functions $c_{a,\rm f}$ in Eq. (2) have been calculated by Rijken and van Neerven in Refs. [2–4] up to the next-to-next-to-leading order (NNLO) for Eq. (1), i.e., the second order in the strong coupling $a_{\rm s} \equiv \alpha_{\rm s}(Q^2)/(4\pi)$. Below we will present the results of a re-calculation of these functions by two approaches differing from that employed in Refs. [2–4].

Besides the second-order coefficient functions, a complete NNLO description also requires the third-order contributions to the splitting functions (so far calculated only up to the second order [5–7]) governing the scale dependence (evolution) of the parton

* Corresponding author.
E-mail address: andreas.vogt@durham.ac.uk (A. Vogt).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2006.05.005

fragmentation distributions. In a notation covering both the (time-like q, $\sigma = 1$) fragmentation distributions and the (space-like q, $Q^2 \equiv -q^2$, $\sigma = -1$) parton distributions, the flavour non-singlet evolution equations read

$$\frac{d}{d\ln Q^2} f_\sigma^{\mathrm{ns}}(x, Q^2) = \int_x^1 \frac{dz}{z} P_\sigma^{\mathrm{ns}}(z, \alpha_s(Q^2)) f_\sigma^{\mathrm{ns}}\left(\frac{x}{z}, Q^2\right) \tag{3}$$

with

$$P_\sigma^{\mathrm{ns}}(x, \alpha_s(Q^2)) = a_s P^{(0)\mathrm{ns}}(x) + a_s^2 P_\sigma^{(1)\mathrm{ns}}(x) + a_s^3 P_\sigma^{(2)\mathrm{ns}}(x) + \cdots. \tag{4}$$

The superscript 'ns' in Eqs. (3) and (4) stands for any of the following three types of combinations of (parton or fragmentation) quark distributions,

$$f_{ik}^{\pm} = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k), \qquad f^{\mathrm{v}} = \sum_{r=1}^{n_f} (q_r - \bar{q}_r), \tag{5}$$

where n_f denotes the number of active (effectively massless) flavours. As detailed below, we have obtained the so far unknown time-like NNLO splitting functions $P_{\sigma=1}^{(2)\mathrm{ns}}(x)$ in Eq. (4).

As already indicated in Eq. (4), the space-like and time-like non-singlet splitting functions are identical at the leading order (LO) [8], a fact known as the Gribov–Lipatov relation. This relation does not hold beyond LO in the usual $\overline{\mathrm{MS}}$ scheme adopted also in this Letter. However, the space-like and time-like cases are related by an analytic continuation in x, as shown in detailed diagrammatic analyses [5,9] at order α_s^2, see also Refs. [10,11]. Moreover, another approach relating the non-singlet splitting functions has been proposed in Ref. [12]. Hence it should be possible to derive time-like quantities from the space-like results computed to order α_s^3 in Refs. [13–15].

We start the analytic continuation from the unrenormalized (and unfactorized) partonic transverse structure function F_1^{b} in deep-inelastic scattering, $\gamma^* q \to X$ (and correspondingly for F_L and $F_3 \to F_A$), calculated in dimensional regularization with $D = 4 - 2\epsilon$ and the scale μ [13–15],

$$F_1^{\mathrm{b}}(a_s^{\mathrm{b}}, Q^2) = \delta(1-x) + \sum_{l=1}^{\infty} (a_s^{\mathrm{b}})^l \left(\frac{Q^2}{\mu^2}\right)^{-l\epsilon} F_{1,l}^{\mathrm{b}}. \tag{6}$$

The bare and renormalized coupling α_s^{b} and α_s are related by (recall $a_s \equiv \alpha_s/(4\pi)$)

$$a_s^{\mathrm{b}} = a_s - \frac{\beta_0}{\epsilon} a_s^2 + \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right) a_s^3 + \cdots \tag{7}$$

with $\beta_0 = 11/3 C_A - 2/3 n_f$ etc. The expansion coefficients in Eq. (6) are then decomposed into form-factor ($\mathcal{F}_l$) and real-emission ($\mathcal{R}_l$, defined analogous to Eq. (6)) contributions [16]

$$\begin{aligned}
F_{1,1}^{\mathrm{b}} &= 2\mathcal{F}_1 \delta(1-x) + \mathcal{R}_1, \\
F_{1,2}^{\mathrm{b}} &= 2\mathcal{F}_2 \delta(1-x) + (\mathcal{F}_1)^2 \delta(1-x) + 2\mathcal{F}_1 \mathcal{R}_1 + \mathcal{R}_2, \\
F_{1,3}^{\mathrm{b}} &= 2\mathcal{F}_3 \delta(1-x) + 2\mathcal{F}_1 \mathcal{F}_2 \delta(1-x) + (2\mathcal{F}_2 + (\mathcal{F}_1)^2)\mathcal{R}_1 + 2\mathcal{F}_1 \mathcal{R}_2 + \mathcal{R}_3.
\end{aligned} \tag{8}$$

The analytic continuation of the form factor to the time-like case is known. The x-dependent functions $\mathcal{R}_l$ are continued from x to $1/x$ [5,9], taking into account the (complex) continuation of q^2 (see Eq. (4.1) of Ref. [16]) and the additional prefactor $x^{1-2\epsilon}$ originating from the phase space of the detected parton in the time-like case [3]. Practically this continuation has been performed using routines for harmonic polylogarithms [17,18] implemented in FORM [19]. The only subtle point in the analytic continuations is the treatment of logarithmic singularities for $x \to 1$, cf. Ref. [9], starting with

$$\ln(1-x) \to \ln(1-x) - \ln x + i\pi. \tag{9}$$

Finally the bare transverse fragmentation function F_T^{b} is re-assembled analogous to Eq. (8), keeping the real parts of the continued $\mathcal{R}_l$ only, and the time-like non-singlet splitting functions and coefficient functions can be read off iteratively from the non-singlet mass factorization formula

$$\begin{aligned}
F_{T,1} &= -\epsilon^{-1} P^{(0)} + c_T^{(1)} + \epsilon a_T^{(1)} + \epsilon^2 b_T^{(1)} + \epsilon^3 d_T^{(1)} + \cdots, \\
F_{T,2} &= \frac{1}{2\epsilon^2} P^{(0)}(P^{(0)} + \beta_0) - \frac{1}{2\epsilon}\left[P_{\sigma=1}^{(1)} + 2P^{(0)} c_T^{(1)}\right] + c_T^{(2)} - P^{(0)} a_T^{(1)} + \epsilon\left[a_T^{(2)} - P^{(0)} b_T^{(1)}\right] + \cdots,
\end{aligned}$$