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Localization of matter and fermion resonances on double walls

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ABSTRACT

We investigate the possibility of localizing various matter fields on the double walls. For spin 0 scalar field, massless zero mode can be normalized on the double walls. However, for spin 1 vector field, the zero mode is not localized on the double walls. In the paper [C.A.S. Almeida, M.M. Ferreira Jr., A.R. Gomes, R. Casana, arXiv:0901.3543 [hep-th]], the authors investigated fermion localization on a Bloch brane, especially, they found fermion resonances on the Bloch brane for both chiralities and related their appearance to branes with internal structure. Inspired by their work, for spin 1/2 spinor field, we focus our attention mainly on the fermion resonances, and also found fermion resonances for both left-handed fermions and right-handed ones on the double walls, which further supports the arguments presented in the paper [C.A.S. Almeida, M.M. Ferreira Jr., A.R. Gomes, R. Casana, arXiv:0901.3543 [hep-th]].

1. Introduction

In recently years, domain walls have received a renewed attention from the physical community after it was pointed out that 4-dimensional gravity can be realized on a thin wall connecting two slices of AdS space [1]. The thin wall construction of Ref. [1] has the disadvantage that the curvature is singular at the location of the wall. This problem can be avoided by coupling gravity to a scalar field. By choosing a suitable potential for the scalar, smooth domain wall (thick domain wall) solutions have been generated in [2,3]. These regularized Randull–Sundrum (RS) walls are in fact particular cases of the more general thick walls – the so-called double walls found in [4]. In the double walls case, the energy density is not peaked around a certain value of the bulk coordinate, but has a double peak instead, representing two parallel walls, or more exactly, a wall with some non-trivial internal structure. It was shown that these double walls can confine gravity [5]. (Similar double walls were found in [6,7] and shown to trap gravity too.)

In the brane world scenarios, an interesting issue is how different observable matter fields of the Standard Model of particle physics are localized on the brane [8–28]. Recently, the authors of Ref. [29] investigated fermion localization on a Bloch brane, especially, they found resonances on the Bloch brane for both chiralities and related their appearance to the fact that the brane has internal structure. One question, which naturally arises, is whether or not fermion resonances are also found on other brane models with internal structure? In this Letter, we investigate the issue localization of various matter fields on the double walls with internal structure constructed in [4], in particular, we focus our attention mainly on the fermion resonances on the double walls. The Letter is organized as follows: In Section 2, we give a quick review of the double wall solution. In Section 3, we discuss localization of various different spin fields on the double walls, we mainly investigate fermion resonances on the double walls numerically. Our summary is presented in Section 4.

2. Review of the double wall solution

Consider the action of Einstein's gravity theory interacting with a real scalar field

$$S=\int d^4x\,dz\sqrt{-g}\left[-\frac{1}{2}R+\frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi-V(\phi)\right], \tag{1}$$

where $g=\det(g_{MN})$, $\det(g_{MN})$ is the metric tensor components with signature $(+,-,-,-,-)$. Capital Latin indices $M,N,\ldots$ run from 0 to 4, R is its Ricci scalar, ϕ is a real scalar field depending on only the extra dimension coordinate z, $V(\phi)$ is a self-interacting

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potential for the scalar field. The coupled Einstein-scalar equations generated from this action has the form

$$R_{MN} - \frac{1}{2} g_{MN} = T_{MN},$$
$$T_{MN} = \nabla_M \phi \nabla_N \phi - g_{MN} \left[\frac{1}{2} \nabla^S \phi \nabla_S \phi + V(\phi) \right],$$
$$\nabla_M \nabla^M \phi = \frac{dV}{d\phi}, \tag{2}$$

where ∇_M is the derivative in g_{MN} and T_{MN} is the stress-energy-tensor. In [4], a solution to (2) was found for the static spacetime of a domain wall with an internal structure. This double wall solution is given by

$$ds_5^2 = e^{2A(z)} \left(\eta_{\mu\nu}\, dx^\mu\, dx^\nu - dz^2 \right),$$
$$A(z) = -\frac{1}{2s} \ln\left[1 + (\alpha z)^{2s}\right], \tag{3}$$

where $\eta_{\mu\nu}$ is the 4-dimensional Minkowski metric, Greek indices $\mu, \nu, \ldots$ run from 0 to 3, α is a real constant and

$$\phi = \phi_0 \arctan\left(\alpha^s z^s\right), \quad \phi_0 = \frac{\sqrt{3(2s-1)}}{s}, \tag{4}$$

with a potential

$$V(\phi) + \Lambda = 3\alpha^2 \sin(\phi/\phi_0)^{2-2/s} \left[\frac{2s+3}{2} \cos^2(\phi/\phi_0) - 2 \right], \tag{5}$$

where $\Lambda = -6\alpha^2$ is the AdS_5 cosmological constant. See [4,5,19] for details.

For $s = 1$ this solution reduces to the regularized of the version of RS thin brane [2,3], while for odd $s > 1$ the energy density is peaked around two values, these walls interpolate between anti-de Sitter asymptotic vacua with $\Lambda = -6\alpha^2$. The wall is in this sense considered a double wall. Thus, our discussion is confined to the case that s is an odd integer and $s > 1$.

3. Localization of various matters

In this section, we study whether various bulk fields with spin ranging from 0 to 1 can be localized on the double walls by means of only the gravitational interaction.

3.1. Spin 0 scalar field

In this section we study localization of a real scalar field on the double walls described in previous section. Let us consider the action of a massless real scalar coupled to gravity:

$$S_0 = \frac{1}{2} \int d^5x \sqrt{-g} g^{MN} \partial_M \Phi \partial_N \Phi, \tag{6}$$

from which the equation of motion can be derived:

$$\frac{1}{\sqrt{-g}} \partial_M \left(\sqrt{-g} g^{MN} \partial_N \Phi \right) = 0. \tag{7}$$

By considering (3) the equation of motion (7) becomes

$$\left[\partial_z^2 + 3(\partial_z A)\partial_z - \eta^{\mu\nu} \partial_\mu \partial_\nu \right] \Phi = 0. \tag{8}$$

The separation of variable is taken as

$$\Phi(x, z) = \sum_n \phi_n(x) \chi_n(z), \tag{9}$$

and demanding $\phi_n(x)$ satisfies the 4-dimensional massive Klein–Gordon equation

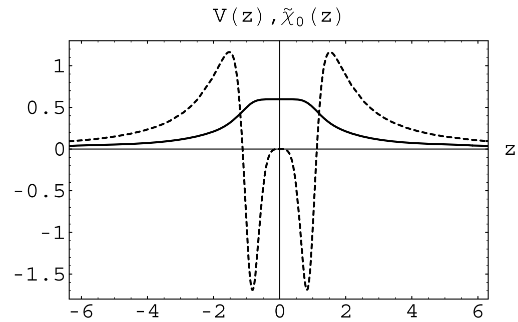

Fig. 1. The dashed line and solid one for the potential (15) and the zero mode (16), respectively. The parameters are set to $\alpha = 1$ and $s = 3$.

$$\eta^{\mu\nu} \partial_\mu \partial_\nu \phi_n(x) = -m_n^2 \phi_n(x), \tag{10}$$

we obtain the equation for $\chi_n(z)$

$$\left[\partial_z^2 + 3(\partial_z A)\partial_z + m_n^2 \right] \chi_n(z) = 0. \tag{11}$$

The 5-dimensional action (6) reduces to the 4-dimensional action for massive scalars, when integrated over the extra dimension under (11) is satisfied and the following orthonormality condition is obeyed

$$\int_{-\infty}^{+\infty} dz\, e^{3A} \chi_m(z) \chi_n(z) = \delta_{mn}. \tag{12}$$

By defining $\tilde{\chi}_n(z) = e^{\frac{3}{2}A} \chi_n(z)$, we get the Schrödinger-like equation

$$\left[-\partial_z^2 + V(z) \right] \tilde{\chi}_n(z) = m_n^2 \tilde{\chi}_n(z), \tag{13}$$

where m_n is the mass of the Kaluza–Klein excitation and the Schrödinger-like potential is given by

$$V(z) = \frac{3}{2} \partial_z^2 A + \frac{9}{4} (\partial_z A)^2. \tag{14}$$

The potential only depends on the warp factor exponent A. For the double walls described in previous section, the potential is reduced to

$$V(z) = \frac{\alpha^2 [15(\alpha z)^{4s-2} - 6(2s-1)(\alpha z)^{2s-2}]}{4[1 + (\alpha z)^{2s}]^2}. \tag{15}$$

For the zero mode ($m_0^2 = 0$), the corresponding wave function takes the form

$$\tilde{\chi}_0(z) = \frac{N_0}{[1 + (\alpha z)^{2s}]^{3/4s}}, \tag{16}$$

where N_0 is a normalization constant. In addition, since this potential has the asymptotic behavior: $V(z = \pm\infty) = 0$, there exists a continuum of states that asymptote to plane waves as $z \to \infty$. The shape of the potential (15) and the zero mode (16) are shown in Fig. 1 for the case $\alpha = 1$ and $s = 3$.

3.2. Spin 1 vector field

Let us start with the action of $U(1)$ vector field:

$$S_1 = -\frac{1}{4} \int d^5x \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS}, \tag{17}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ as usual. From this action the equation motion is given by