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Can relic superhorizon inhomogeneities be responsible for large-scale CMB anomalies?

Xian Gao

Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, No. 55, Zhong-Guan-Cun East Road, Hai-Dian District, Beijing 100080, PR China

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ABSTRACT

We investigate the effects of the presence of relic classical superhorizon inhomogeneities during inflation. This superhorizon inhomogeneity appears as a gradient locally and picks out a preferred direction. Quantum fluctuations on this slightly inhomogeneous background are generally statistical anisotropic. We find a quadrupole modification to the ordinary isotropic spectrum. Moreover, this deviation from statistical isotropy is scale-dependent, with a $\sim -1/k^2$ factor. This result implies that the statistical anisotropy mainly appears on large scales, while the spectrum on small scales remains highly isotropic. Moreover, due to this $-1/k^2$ factor, the power on large scales is suppressed. Thus, our model can simultaneously explain the observed anisotropic alignments of the low-ℓ multipoles and their low power.

1. Introduction

The current observation data support the standard ΛCDM model [1]. Recently, however, there is an growing interest in analyzing possible large-scale anomalies of CMB, from both theoretical and observational sides [2–8,11–13].

It appears that the lowest CMB multipoles are anomalous in two seemingly distinct aspects. Firstly, the angular power C_ℓ at the lowest ℓ is abnormally suppressed [3]. Secondly, they have an improbable directionality revealed by the fact that for a certain preferred orientation, one m-mode absorbs most of the power, which may imply the presence of an "axis of evil" [4,5].

The statistical properties of perturbations carry the same symmetries as the background on which they are generated. In the standard scenario, quantum fluctuations are assumed to be generated on a spatially homogeneous and isotropic background. Background spatial inhomogeneities are neglected in most of the analysis. Indeed, a long period of inflation pulls all non-smooth classical initial conditions out of the horizon. However, if inflation lasts the minimal number 60 e-folds or so, perturbations with comoving wave-numbers of cosmological interest cross the horizon just during the earliest several e-folds of inflation. Thus, the relic inhomogeneities may leave some marks in the primordial quantum fluctuations.

In this Letter, we investigate the possible effects of a relic classical superhorizon inhomogeneity during inflation, especially its effects on the statistical properties of quantum fluctuations. In [6,7], a single superhorizon perturbation mode were considered to explain the power asymmetry of CMB on large scales. While in this work, we consider relic superhorizon inhomogeneities as *background*. Perturbation theory in the presence of a spatially inhomogeneous inflaton background value has been investigated by several authors before [11,12]. Quantum fluctuations on this slightly inhomogeneous background are generally statistical anisotropic.

The deviations from statistical isotropy can take on many forms, which may correspond to different physical origins. One simple form was presented in [4], $P(\boldsymbol{k}) = P(k)[1 + g(k)(\hat{\boldsymbol{k}}\cdot\boldsymbol{n})^2]$. In this work, we find that the (leading-order) correction to the statistically isotropic power spectrum is of the ACW form [4], but with a k-dependent factor $g(k) \sim -1/k^2$. Thus, the power spectrum deviates from isotropy mainly on large scales (small k), while remains highly isotropic on small scales. Moreover, the spectrum itself is also suppressed on large scales. Thus, our model can simultaneously explain the observed anisotropic alignments of the low-ℓ multipoles and their low power.

2. Model and background

In this work we investigate general single scalar field inflationary models, with an action of the form $S = \int d^4x\sqrt{-g}\,[R/2 + P(X,\phi)]$, where $X = -\frac{1}{2}(\partial\phi)^2$. Now we consider a slightly inhomogeneous inflaton background $\bar{\phi}(t,\boldsymbol{x})$. Firstly, we assume the deviation from homogeneity is very small, i.e., if $\bar{\phi}(t)$ is a spatial average of $\bar{\phi}(t,\boldsymbol{x})$ (e.g. in one Hubble volume), then we assume that

E-mail address: gaoxian@itp.ac.cn.

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$$\left|\frac{\bar{\phi}(t,\boldsymbol{x})-\bar{\phi}(t)}{\bar{\phi}(t)}\right| \ll 1. \tag{2.1}$$

Secondly, we assume the inhomogeneities are superhorizon. In other words, the typical comoving scale of these background inhomogeneities l is much larger than today's comoving Hubble scale, $l \gg (a_0H_0)^{-1}$. Actually, it has been known long before that inflation can occur in the presence of superhorizon initial inhomogeneities [9]. In other words, the inflaton field initially should be smooth up to physical scales larger than $\sim H^{-1}$. Thus, today's observational universe can indeed inflate from an initial small patch with classical inhomogeneities with typical comoving scale $l \gg (a_0H_0)^{-1}$.

Superhorizon inhomogeneities locally look like a gradient, $\bar{\phi}(t,\boldsymbol{x}) = \bar{\phi}(t,\boldsymbol{x}_0) + (\boldsymbol{x}-\boldsymbol{x}_0)\cdot\nabla\bar{\phi}(t,\boldsymbol{x}_0)+\cdots$. In this work, we neglect higher-order derivatives and treat $\nabla\bar{\phi}$ as approximately constant (while $\bar{\phi}$ itself is indeed slow-rolling, for instance).

3. Linear perturbations

We split $\phi(t,\boldsymbol{x})$ into background and fluctuation configurations,

$$\phi(t,\boldsymbol{x}) = \bar{\phi}(t,\boldsymbol{x}) + \delta\phi(t,\boldsymbol{x}). \tag{3.1}$$

In the following we denote $\partial_i\bar{\phi}(t,\boldsymbol{x}) = A_i$, which we have assumed to be constant.

As a first-step investigation, we neglect the metric perturbations. Thus the calculation is straight forward since all we have to do is to expand the action for the scalar field directly. According to (3.1), simple Taylor expansion of $P(X,\phi)$ around the background $\bar{\phi}$ up to second order of $\delta\phi$ gives $-\frac{1}{2}\Sigma^{\mu\nu}\partial_\mu\delta\phi\partial_\nu\delta\phi - P_{,X\phi}\partial^\mu\bar{\phi}\partial_\mu\delta\phi\delta\phi + \frac{1}{2}P_{,\phi\phi}\delta\phi^2$, where we have defined

$$\Sigma^{\mu\nu} \equiv P_{,X}g^{\mu\nu} - P_{,XX}\partial^\mu\bar{\phi}\partial^\nu\bar{\phi}. \tag{3.2}$$

Note that due to the non-vanishing background gradient $\partial_i\bar{\phi}$, Σ^{ij} is not proportional δ_{ij} any more. The presence of the additional term which is proportional $\partial_i\bar{\phi}\partial_j\bar{\phi}$ in Σ^{ij} breaks spatial rotational invariance and will be responsible for the generation of statistical anisotropies of the scalar perturbations $\delta\phi$.

After introducing a new variable $u(\eta,\boldsymbol{x}) = a\sqrt{-\Sigma^{00}}\delta\phi$, the second-order action for the scalar field perturbations can be written as

$$S = \int d\eta \frac{d^3k}{(2\pi)^3}\frac{1}{2}\left[u'_{-\boldsymbol{k}}u'_{\boldsymbol{k}} + \frac{2i\sqrt{\epsilon}\gamma}{aH}(\boldsymbol{A}\cdot\boldsymbol{k})u'_{-\boldsymbol{k}}u_{\boldsymbol{k}} - \left(c^2k^2 - \frac{a''}{a} + \mathcal{M} + \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{a^2H^2}\right)u_{-\boldsymbol{k}}u_{\boldsymbol{k}}\right], \tag{3.3}$$

where η is comoving time, and a prime represents $\partial/\partial\eta$, $\mathcal{H} \equiv a'/a$, and $\mathcal{M}$ is an effective mass term, ϵ is defined as $\epsilon = \dot{\phi}^2/H^2$. Here we define two dimensionless parameters

$$c^2 = \frac{P_{,X}}{-\Sigma^{00}}, \qquad \gamma = \frac{H^2P_{,XX}}{\Sigma^{00}}. \tag{3.4}$$

In this Letter, we assume $\gamma > 0$. Note that the canonical case corresponds to $c^2 = 1$ and $\gamma = 0$. In deriving (3.3), we use the approximation that $\Sigma^{\mu\nu}$ is a function of only η, that is, we neglect the spatial dependence of $\Sigma^{\mu\nu}$.

3.1. Equation of motion for the perturbations and solutions

The classical equation of motion for the mode function according to the second-order action (3.3) is

$$u''_{\boldsymbol{k}} + \frac{2i\sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{aH}u'_{\boldsymbol{k}} + \left(c^2k^2 - \frac{a''}{a} + \mathcal{M}a^2 + \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{a^2H^2}\right)u_{\boldsymbol{k}} = 0. \tag{3.5}$$

As it can be seen, an anisotropic dispersion relation arises, which will be responsible for the generation of a statistically anisotropic power spectrum.

Now we take the scale factor as $a(\eta) = -1/H\eta$. Remarkably, in this simplest case, Eq. (3.5) has an analytic solution,

$$u_{\boldsymbol{k}}(\eta) = \frac{\Gamma(\alpha-\nu)}{2^{\nu+1}\sqrt{\pi}}e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}e^{\frac{i}{2}\lambda\eta^2}\sqrt{-\eta}(-ck\eta)^\nu U(\alpha,\nu+1,z), \tag{3.6}$$

in which $U(\alpha,\nu+1,z)$ is the confluent hypergeometric function, with

$$\begin{aligned}
\nu &= \sqrt{\frac{9}{4} - \frac{\mathcal{M}}{H^2}}, \\
\alpha &= \frac{1}{2}(\nu+1) - \frac{ic^2k^2 - \sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{4|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}}, \\
z &= -i|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}\eta^2, \\
\lambda &= |\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2} + \sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k}).
\end{aligned} \tag{3.7}$$

Due to the presence of the factor $\boldsymbol{A}\cdot\boldsymbol{k}$, the modes generally depend on $\boldsymbol{k}$ rather than $k = |\boldsymbol{k}|$. Here the coefficient in (3.6) is chosen in order to satisfy the Wronskian normalization condition $u'_{\boldsymbol{k}}(\eta)u^*_{\boldsymbol{k}}(\eta) - u'^*_{\boldsymbol{k}}(\eta)u_{\boldsymbol{k}}(\eta) = i$. In the limit $A_i \to 0$, it can be verified that (3.6) reduces to the well-known functional form $u_{\boldsymbol{k}}(\eta) \xrightarrow{A_i\to 0} \frac{\sqrt{\pi}}{2}e^{\frac{i\pi}{2}(\nu+\frac{1}{2})}\sqrt{-\eta}H^{(1)}_\nu(-ck\eta)$, which describes nothing but the normalized mode function of a massive scalar field in pure de Sitter spacetime. Thus our mode solution (3.6) generalizes the standard homogeneous and isotropic background to the case of the presence of superhorizon background inhomogeneities.

3.2. Anisotropic power spectrum

When all modes of cosmological interest exit the Hubble scale, that is, in $\eta \to 0$ limit, since $U(\alpha,\beta,z) \to \frac{\Gamma(\beta-1)}{\Gamma(\alpha)}z^{1-\beta}$ as $z \to 0$ (when $\beta > 2$), we get

$$u_{\boldsymbol{k}}(\eta) = \mathcal{A}(\boldsymbol{k})e^{\frac{i\pi}{2}(\nu-\frac{1}{2})}2^{\nu-\frac{3}{2}}\frac{\Gamma(\nu)}{\Gamma(3/2)}\frac{1}{\sqrt{2ck}}(-ck\eta)^{\frac{1}{2}-\nu}, \tag{3.8}$$

where we have defined an anisotropic deformation factor,

$$\mathcal{A}(\boldsymbol{k}) \equiv \frac{\Gamma(\alpha-\nu)}{\Gamma(\alpha)}\left(\frac{4i|\boldsymbol{A}\cdot\boldsymbol{k}|\sqrt{\gamma+\epsilon\gamma^2}}{c^2k^2}\right)^{-\nu}, \tag{3.9}$$

which is responsible for the anisotropic deformation of the power spectrum on large scales. Here α, ν are given in (3.7). In the standard scenarios, $\mathcal{A}(\boldsymbol{k})$ is just $\mathcal{A}(\boldsymbol{k}) = 1$. In our case, $\mathcal{A}$ depends on $\boldsymbol{k}$, not only on its amplitude k, but also on it direction, more precisely, on $\hat{\boldsymbol{A}}\cdot\hat{\boldsymbol{k}}$ (see Fig. 1).

We would like to investigate the leading order effect of $A_i = \partial_i\bar{\phi}$. Taking the limit $A_i \to 0$ in (3.9) and using Stirling's formula, we get

$$\mathcal{A}(\boldsymbol{k}) = 1 - i\frac{\nu\sqrt{\epsilon}\gamma(\boldsymbol{A}\cdot\boldsymbol{k})}{c^2k^2} - \frac{\gamma(\boldsymbol{A}\cdot\boldsymbol{k})^2}{c^4k^4}\left[\frac{2}{3}\nu(\nu^2-1) + \frac{\nu(\nu+1)(4\nu-1)}{6}\epsilon\gamma\right]. \tag{3.10}$$