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Shock treatment: Heavy quark drag in a novel AdS geometry

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ABSTRACT

We calculate the drag force on a heavy quark hit by a shock wave, thus generalizing the strongly coupled AdS/CFT heavy quark drag calculations to both hot and cold nuclear matter. The derivation employs the trailing string configuration, similar to that used in the literature for a quark moving through a thermal medium, though in the shock metric the string profile is described by a much simpler analytic function. Our expression for the drag depends on the typical transverse momentum scale of the matter in the shock. For a thermal medium this scale becomes proportional to the temperature, making our drag coefficient and momentum limit of applicability identical to those found previously. As the shock wave can be composed of either thermalized or non-thermalized media, our derivation extends the existing drag calculations to the case of arbitrarily distributed matter.

1. Introduction

The simultaneous emergence of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1–4] as a tool to study strongly coupled systems (see [5,6] for reviews) and the failure of perturbative QCD (pQCD) techniques [7] to quantitatively describe the observed heavy ion physics phenomena has inspired both a critical reevaluation of the traditional methods and an explosion of new research [8–13]. Previous AdS/CFT calculations [9–12,14–18] of the heavy quark drag assumed a strongly coupled thermal medium of $\mathcal{N}=4$ super-Yang–Mills (SYM) plasma. In this Letter we extend the earlier work on the heavy quark drag in a thermal medium to that of an arbitrary medium; in particular our formalism applies to both hot and cold strongly coupled nuclear matter.

The AdS/CFT conjecture postulates a duality between certain field theories and the compactification of Type IIB string theory in various geometries [1–4]. In particular $\mathcal{N}=4$ SYM theory is dual to Type IIB string theory in $\mathrm{AdS}_5 \times S^5$. What makes the conjecture so useful, but also so difficult to prove, is that the weak coupling limit of one theory is dual to the strong coupling limit of the other. Of especial interest to the heavy ion community is the strong coupling limit of QCD, where the only previous theoretical tool was numerical lattice simulations. While lattice simulations remain the only method to obtain quantitative results for strongly-coupled QCD, one may argue that for some observables similarities between QCD and $\mathcal{N}=4$ SYM theory can be exploited to improve our qualitative understanding of the former by performing calculations in the latter. In the limit of large 't Hooft coupling and number of colors N_c, the $\mathcal{N}=4$ SYM theory is dual to an (often) analytically tractable theory, the classical limit of string theory: classical supergravity (SUGRA). Despite the many differences between QCD and $\mathcal{N}=4$ SYM, there have been a number of qualitative successes in applying the AdS/CFT ideas to heavy ion phenomenology at RHIC (see, e.g., [5,6] for a review). On the other hand, from both experimental and theoretical standpoints, perturbative methods as applied to jet quenching physics should be viewed with increased skepticism. For a brief review of some of the disparities seen in comparisons of phenomenological theory and data see, e.g., [19]. From a self-inconsistency standpoint the assumption of a small coupling, $g \ll 1$, in high-p_T energy loss has never been true—at RHIC energies and temperatures $g \sim 2$—with the momentum scale, set by either the Gyulassy–Wang model [20] of Yukawa-like scattering centers or the saturation scale, on the order of 0.5–1 GeV $\sim \mathcal{O}(1)\Lambda_{\mathrm{QCD}}$, far from perturbative.

An understanding of the limits of applicability, of the inherent theoretical error, are required in order to either gain confidence in or falsify a calculation with experimental data. In deep inelas-

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tic scattering (DIS) and Drell–Yan production, the rigorous framework of factorization allows for a well-controlled pQCD expansion (see [21] and references therein). For heavy ion collisions k_T-factorization does not hold [22], and there is no proof of collinear factorization to all twists. The AdS calculations are under even less control. Until a dual string geometry is found for QCD one cannot quantitatively estimate the changes in observables from using a different theory; there is no path through dual theories approaching QCD whose parameterization may be used as an expansion parameter.

What one can do is explore the results from different theories and geometries in an attempt to discover universal behavior. This may give one reason to believe that the result will hold for QCD, at least at the qualitative level. In this Letter we follow this strategy by considering the heavy quark momentum loss in a novel geometry. Previous drag force calculations focused on motion of the gauge theory quark in vacuum (empty AdS metric) [14,16,17], or in a static thermal medium (black hole and black hole-like metrics) [9–12,15,16]. Here we examine the drag force on a static quark inside a shock wave. In particular the shock can model the dense medium produced in a heavy ion collision. Equivalently it can model a nucleus incident on a probe quark, giving rise to cold-matter energy loss as is often considered in proton–nucleus collisions. We show how the shock may be composed of any distribution of matter, either thermalized or not: due to Lorentz time-dilation the shock wave may represent a "snapshot" of any non-equilibrium matter. Hence our result generalizes the drag force of [10–12] to the case of a non-thermal medium. We also study the limitations of our calculation and find that the momentum "speed limit" of applicability for the shock wave calculation is parametrically exactly the same as for the black hole metric.

2. Shock metric, EOM, and the trailing string solution

We will consider the generalized "shock" metric [13]

$$ds^2 \equiv G_{\mu\nu}\, dx^\mu\, dx^\nu$$
$$= \frac{L^2}{z^2}\left[-2\,dx^+\,dx^- + 2\mu z^4\theta(x^-)\,dx^{-2} + dx_\perp^2 + dz^2\right] \tag{1a}$$
$$= \frac{L^2}{z^2}\Big[-\big(1-\mu z^4\theta(x^-)\big)\,dt^2 - 2\mu z^4\theta(x^-)\,dt\,dx + \big(1+\mu z^4\theta(x^-)\big)\,dx^2 + dx_\perp^2 + dz^2\Big], \tag{1b}$$

where we have used the $x^\pm = (t \pm x)/\sqrt{2}$ normalization of light-cone coordinates with $x = x^3$ and dropped the $d\Omega_5^2$ standard metric of the five-sphere in $\mathrm{AdS}_5 \times S^5$. As usual L is the radius of the S^5 space. Also $dx_\perp^2 = (dx^1)^2 + (dx^2)^2$ is the transverse part of the metric, and $\mu, \nu = 0, \ldots, 4$.

The physical justification for describing Eq. (1a) as a shock metric comes from the application of holographic renormalization [23], which relates the metric in Fefferman–Graham form [24] in $d+1$ dimensions to the energy–momentum tensor in the d-dimensional boundary theory. For our particular choice of metric, Eq. (1a), the above prescription yields

$$\langle T_{--}\rangle = \frac{N_c^2}{\pi^2}\mu\theta(x^-), \tag{2}$$

which describes an ultrarelativistic shock wave (shock front) moving in the positive $x^3 = x$ direction.

Previous calculations used $\mu\delta(x^-)$ as the coefficient of the dx^{-2} term in Eq. (1a) to represent the Lorentz-contracted pancake of nuclear matter probed in DIS [25] or seen in heavy ion collisions [26]. Note that in those papers μ is a slightly different quantity with units GeV^3, unlike GeV^4 here. As observed in [13] the coefficient of dx^{-2} can be any function of x^-; the resulting metric still satisfies Einstein's equations. We take it $\propto \theta(x^-)$ to represent an incoming dense medium of nuclear matter colliding with a heavy quark in its rest frame. The setup is shown in Fig. 1, where the shaded region is the incoming shock wave. The fifth dimension z increases as one moves down the z axis: the top of the diagram is the boundary of AdS_5 space ($z = 0$); at the bottom ($z = \infty$) is the stack of N_c D3 color branes. A finite mass fundamental representation heavy quark in the 4D field theory corresponds in the 10-dimensional supergravity theory to the endpoint of an open Nambu–Goto string terminating on a D7 brane; the other end of the open string ends on the N_c stack of D3 branes at $z = \infty$. The D7 brane wraps an $S^3 \subset S^5$ and fills the asymptotically AdS space from $z = 0$ down to $z = z_M$ [27]. Before the collision the string hangs straight down (left panel of Fig. 1), while after the collision the string trails behind the quark (right panel of Fig. 1).

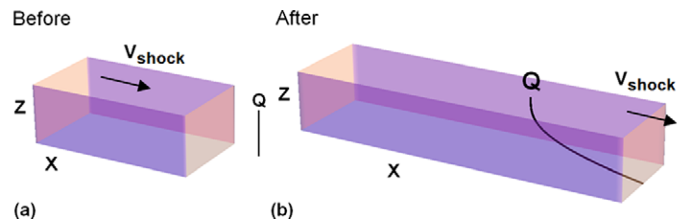

Fig. 1. Cartoon of the shock medium colliding with the heavy quark Q in its rest frame. (a) Before the collision the string representing the heavy quark hangs from the D7 brane straight down to the D3 branes at the origin. (b) After the collision the string trails behind Q. As the shock is moving in the positive direction ($v_{\mathrm{sh}} > 0$) in the heavy quark rest frame momentum actually flows up the string as the shock tries to transfer positive momentum to the quark; see Section 3 for more details.

Ordinarily the spatial direction picked out when using light cone coordinates represents the beam direction. That would be the case here if we apply our model to the description of energy loss in proton–nucleus collisions, i.e., in cold nuclear matter. For jet energy loss in heavy ion collisions x corresponds to the direction of motion of the heavy quark in the lab frame, an orientation often taken transverse to the beam.

While [10] extended the heavy quark drag calculations to more general metrics of the black hole type, a novel feature of the $G_{\mu\nu}$ used here is its lack of an event horizon. If light can pass through a surface coming from inside the suspected black hole, the surface is not a true horizon. For our metric a light ray moving towards the boundary of AdS space both in the z and x direction can cross the suspected horizon at $z_h = \mu^{-1/4}$. Since the light ray can therefore escape from inside the suspected black hole, the surface at $z_h = \mu^{-1/4}$ is not a true horizon.

We are interested in the motion of a heavy quark in the background specified by the metric of Eq. (1b). The test string action is

$$S_{\mathrm{NG}} = -T_0 \int d\tau\, d\sigma\, \sqrt{-g},$$
$$g = \det g_{ab}, \quad g_{ab} = G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu, \tag{3}$$

where $G_{\mu\nu}$ is the spacetime metric of Eq. (1b), Greek indices refer to spacetime coordinates, and Latin indices to worldsheet coordinates. $X^\mu = X^\mu(\sigma)$ specifies the mapping from the string worldsheet coordinates σ^a to spacetime coordinates x^μ. The backreaction of the fundamental string ($\mathcal{O}(N_c)$) is neglected as compared to the $\mathcal{O}(N_c^2)$ contributions from the adjoint fields of $\mathcal{N} = 4$ SYM [28].

Varying the action, Eq. (3), yields the equations of motion: