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On the problem of vacuum energy in brane theories

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ABSTRACT

We point out that modern brane theories suffer from a severe vacuum energy problem. To be specific, the Casimir energy associated with the matter fields confined to the brane, is stemming from the one and the same localization mechanism which forms the brane itself, and is thus generically unavoidable. Possible practical solutions are discussed, including in particular spontaneously broken supersymmetry, and quantum mechanically induced brane tension.

The idea that our universe is a brane embedded in a higher dimensional space–time has received a great deal of attention for several reasons. First and foremost, quantum gravity seems to demand it, and to that argument joins superstring/M theory which predicts ten/eleven dimensions of space–time. Brane gravity has made some remarkable progress over the last few years, dynamical localization mechanisms have been found, and many 4-dim general relativistic results have been reproduced [1–4].

The most fundamental and important aspect of brane theory is that although we live in a high dimensional space (the bulk), all Standard Model fields are localized on a 4-dim hypersurface (the brane) with some finite thickness δ. This brane thickness is often taken to be zero for simplifying calculations, but in all realistic models, especially those which include quantum corrections, this thickness must be finite. The limits on 4-dim gravity at low scales are fairly loose. We know that gravity is 4-dim to about 10 microns, and different brane models make use of that loose limit. Standard Model fields, however, are much more confined. And since no accelerator ever detected signatures that can be interpreted as higher dimensional propagation, one deduces that $\delta \leqslant (1\ \text{TeV})^{-1}$. This upper bound is completely independent of the brane model at hand, and will be the main source of the issue discussed in this Letter. To be precise, it will be translated into a lower bound on the "residual" vacuum energy on the brane, to be regarded as an unavoidable outcome of brane gravity.

The vacuum energy problem is still an open question. According to quantum field theory (QFT), summing the zero-point energies of all normal modes of matter fields up to the Planck scale (or even the QCD cutoff) gives rise to enormous energy density of the vacuum around us. Despite this, no such energy density seems to exist (dark energy resembles such an energy density, but its observed value is inexplicably smaller than QFT predicts). However, we also know that vacuum energy does exist in some form because effects originated from vacuum fluctuations have been predicted and measured. One such effect, and perhaps the most direct observation of vacuum energy, is the Casimir effect [5,6]. In general terms, the Casimir effect is the variation in vacuum energy caused by the addition of boundary conditions to the system. A quantum field subject to boundary conditions (caused by other matter fields or strongly curved space–time) would have a different vacuum energy than a free field. The difference between the vacuum energies of the constrained and free field is the Casimir energy (note that the Casimir energy is independent of the QFT cutoff). In fact, it is the quantum backreaction of the field to the boundary conditions. A concrete example of such effect is the attractive force

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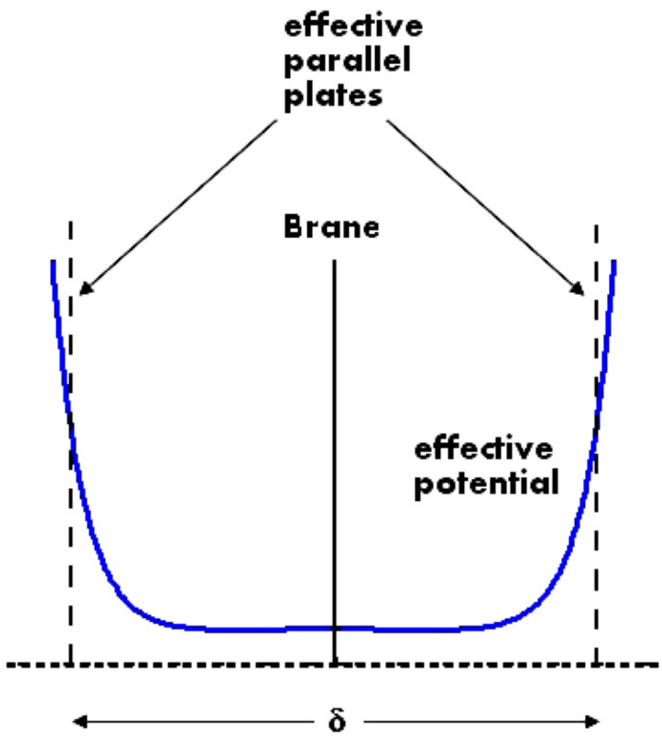

Fig. 1. Effective potential: In brane scenario, the Standard Model fields are localized around a single brane. This can be described via an effective localization potential with a very sharp minima. The latter can be approximated by a potential well with a certain width δ.

between two parallel conductive plates in a vacuum. The plates create boundary conditions for the electro-magnetic field, and thus a force is generated [5].

In brane theory this means that the localization of Standard Model fields on the brane, regardless of the underlying mechanism [7], results in essentially the same effect as the one caused by a pair of conducting plates (Fig. 1). Several papers dealing with the Casimir effect in brane gravity have already been presented [8–11], but most of them focus on the Casimir effect between two branes. Unlike the latter, the present work is model independent, and deals with the Casimir effect generated by the familiar matter fields embedded on a single brane. Once these fields get localized to the brane, their confinement is analogous to the confinement of an electro-magnetic field between conducting plates. The form of the exact localization mechanism is unimportant, as it will always result in the same energy up to a constant of $\mathcal{O}(1)$.

The Casimir energy of two plates in $4+n$ dimensions with a separation δ,[1] is given by [5]

$$E = \frac{\eta \hbar A}{\delta^{3+n}}, \tag{1}$$

where A is the area of the plates (hyper area, in the general case) and η is a constant. From this point on, we will use the notation $\hbar = 1$. The value of η depends on n, on the exact form of the localization potential, and on the number of Standard Model fields (degrees of freedom). The energy density between the plates is therefore

$$\rho_{\text{bulk}} = \frac{\eta}{\delta^{4+n}}. \tag{2}$$

Since we are talking about a D3 brane, and assume more than one extra-dimension, we do not face a parallel plate system, but rather, cylindrical or spherical boundaries. While leaving the form of Eq. (2) intact, this will accordingly modify the value of η. Eq. (2) describes the bulk energy density. The energy density on the brane is obtained by integrating out all n extra-dimensions, so that

$$\rho = \frac{\tilde{\eta}}{\delta^4}, \tag{3}$$

with $\tilde{\eta} \neq \eta$. This is a *constant* energy density on the brane, thus it is a direct contribution to the cosmological constant.[2] The exact value of $\tilde{\eta}$ is of course model dependent, but it cannot deviate too much from $\mathcal{O}(1)$. With this in mind, taking into account the experimental bound on δ, we evaluate the energy density from Eq. (3) and find

$$\rho \geqslant (1\ \text{TeV})^4. \tag{4}$$

This is to be contrasted with the much smaller value of the cosmological constant (dark energy) $\rho_\Lambda \sim (10^{-3}\ \text{eV})^4$, leading to a 60 orders of magnitude discrepancy. Unlike the ordinary vacuum energy, this energy cannot be 'swept under the carpet' because it does not stem directly from the action, but rather, caused by (quantum corrections due to) the abnormal structure of matter and space–time. This constitutes a serious problem. In order for brane theories to be realistic, one must find a way to cancel or suppress this energy.

It is natural to turn first to supersymmetry, the natural cure for vacuum energy [12]. If unbroken, SUSY assures an absolute cancellation of the vacuum energy. However, we know that SUSY must be broken at an energy scale $M_{\text{SUSY}} \geqslant 1$ TeV. If SUSY is broken at a much lower energy scale than the localization energy, then we might expect a strong suppression of the Casimir energy [13]. As a simple example, let us consider the case of a scalar field and its superpartner "calar". In order to perform exact calculations, we assume one extra-dimension ($n = 1$) and simplify the localization potential to be an infinite well of width δ. The Casimir energy density generated by the scalar field on the brane is given by [5]

$$\rho_{\text{scalar}} = -2\left(\frac{m}{4\pi}\right)^{5/2} \frac{1}{\delta^{3/2}} \sum_{j=1}^{\infty} \frac{K_{5/2}(2m\delta j)}{j^{5/2}}, \tag{5}$$

where $K_\nu(x)$ is the modified Bessel function of the second type, and m is the mass of the scalar field. If SUSY is unbroken, the "calar" field will give the exact same result but with an opposite sign. However, if SUSY is broken, even at an energy scale lower than δ^{-1}, then the masses will be slightly corrected, such that $m^2_{\text{scalar}} - m^2_{\text{calar}} = \Delta m^2$. In that case, and assuming $M_{\text{SUSY}} \gg m \gg \Delta m$, the residual Casimir energy density on the brane becomes

$$\rho \cong \lim_{m \to 0} \frac{\Delta m^2}{2m} \frac{\partial \rho_{\text{scalar}}}{\partial m}, \tag{6}$$

where $\rho = \rho_{\text{scalar}} + \rho_{\text{calar}}$. Evaluating Eq. (6) using Eq. (5), we obtain

$$\rho = \frac{\zeta(3)}{64\pi^2}\left(\frac{\Delta m}{\delta}\right)^2. \tag{7}$$

[1] We assume all Standard Model fields are localized in the same way. We also assume for simplicity, that all masses are much lower than the localization scale δ^{-1}, so we may treat them as massless. Both assumptions may not be exact but they greatly simplify the equations and deviating from these assumptions will not modify our conclusions at all.

[2] We note that brane theories can bring forward other contributions to the cosmological constant. However, the Casimir energy is the only model independent contribution.