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Relaxing a large cosmological constant

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ABSTRACT

The cosmological constant (CC) problem is the biggest enigma of theoretical physics ever. In recent times, it has been rephrased as the dark energy (DE) problem in order to encompass a wider spectrum of possibilities. It is, in any case, a polyhedric puzzle with many faces, including the cosmic coincidence problem, i.e. why the density of matter ρ_m is presently so close to the CC density ρ_Λ. However, the oldest, toughest and most intriguing face of this polyhedron is the big CC problem, namely why the measured value of ρ_Λ at present is so small as compared to any typical density scale existing in high energy physics, especially taking into account the many phase transitions that our Universe has undergone since the early times, including inflation. In this Letter, we propose to extend the field equations of General Relativity by including a class of invariant terms that automatically relax the value of the CC irrespective of the initial size of the vacuum energy in the early epochs. We show that, at late times, the Universe enters an eternal de Sitter stage mimicking a tiny positive cosmological constant. Thus, these models could be able to solve the big CC problem without fine-tuning and have also a bearing on the cosmic coincidence problem. Remarkably, they mimic the ΛCDM model to a large extent, but they still leave some characteristic imprints that should be testable in the next generation of experiments.

1. Introduction

High Energy Physics is described by quantum field theory (QFT) and string theory. Unfortunately, these theoretical descriptions are plagued by large hierarchies of energy scales associated to the existence of many possible vacua. Such situation is at the root of the old and difficult CC problem [1], i.e., the formidable task of trying to understand the enormous ratio between the theoretical computation of the vacuum energy density and its observed value, $\rho_\Lambda^0 \sim 10^{-47}$ GeV^4, obtained from modern cosmological data [2]. The extremal possibility occurs when the Planck mass $M_P \sim 10^{19}$ GeV is used as the fundamental scale; then the ratio M_P^4/ρ_Λ^0 becomes $\sim 10^{123}$. One may think that physics at the Planck scale is not well under control and that this enormous ratio might be fictitious. However, consider the more modest scale $v = 2M_W/g \simeq 250$ GeV of the electroweak Standard Model (SM) of Particle Physics (the experimentally most successful QFT known to date), where M_W and g are the $W^\pm$ boson mass and $SU(2)$ gauge coupling, respectively. In this case, that ratio reads $|\langle V\rangle|/\rho_\Lambda^0 \gtrsim 10^{55}$, where $\langle V\rangle = -(1/8)M_H^2 v^2 < 0$ is the vacuum energy (i.e. the expectation value of the Higgs potential) and $M_H \gtrsim 114.4$ GeV is the lower bound on the Higgs boson mass. Although one may envisage the possibility that there is a cancelation between the various theoretical contributions to the physical CC (including the bare value), this has never been considered a realistic option owing to the enormous fine-tuning that it entails (which, in addition, must be corrected order by order in perturbation theory).

In this Letter, we discuss a dynamical mechanism that protects the Universe from any initial CC of arbitrary magnitude $|\rho_\Lambda^i| \gg \rho_\Lambda^0$, which could emerge, for instance, from quantum zero-point energy (contributing roughly $\sim m^4$ for any mass m), phase transitions ($\rho_\Lambda^i = \langle V\rangle$) or even vacuum energy at the end of inflation. We admit that $\rho_\Lambda = \rho_\Lambda(t)$ (with $\rho_\Lambda(t_i) = \rho_\Lambda^i$, $\rho_\Lambda(t_0) = \rho_\Lambda^0$) can actually be an effective quantity evolving with time.

Phenomenological models with variable ρ_Λ have been considered in many places in the literature and from different perspectives, see e.g. [3]. At the same time, models with variable CC with a closer relation to fundamental aspects of QFT have also been proposed [4–7]. In all these cases, the effective quantity $\rho_\Lambda = \rho_\Lambda(t)$ still has an equation of state (EOS) $p_\Lambda = -\rho_\Lambda$ and, in this sense, it can be called a CC term.

The basic framework of our proposal is the generalized class of ΛXCDM models introduced in [8], in which there is a fixed or variable ρ_Λ term together with an additional "effective" component X (in general *not* related to a fundamental, e.g. scalar, field).

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This particular class of variable CC models is especially significant in that they could cure the cosmic coincidence problem [8] in full consistency with cosmological perturbations [9]. Here we present a generalization of these models that might even cure the old ("big") CC problem [1]. Recently, in [10] a model along these lines was introduced with a DE density ρ_D and an inhomogeneous EOS $p_D = \omega\rho_D - \beta H^{-\alpha}$ which includes a term proportional to the negative power of the Hubble rate H. This additional term becomes sufficiently large to compensate an initial ρ_Λ^i when this is about to dominate the universe and forces it eventually into a final de Sitter era with a small CC. For recent related work on relaxation mechanisms, see e.g. [11–13]. In a different vein, the CC problem can also be addressed in quantum cosmology models of inflation, through the idea of multiuniverses [14] and the application of anthropic considerations [1].

Let us recall that, historically, most of the models addressing the relaxation of the CC were based on dynamical adjustment mechanisms involving scalar field potentials [15]. In the present work, the relaxation mechanism that we propose is also dynamical, it does not require any fine-tuning and, as noted, it does not depend in general on scalar fields. To be more precise, the model we present here is a ΛXCDM relaxation model of the CC, which includes also matter and radiation eras. We study the two possibilities $\rho_\Lambda^i < 0$ and $\rho_\Lambda^i > 0$, with arbitrary value. For $\rho_\Lambda^i < 0$, our scenario avoids the big crunch at early times and allows the cosmos to evolve starting from a radiation regime with subsequent matter and de Sitter eras like the standard ΛCDM model. Finally, let us emphasize that our method to tackle the CC problem is formulated directly at the level of the (generalized) field equations, rather than from an effective action functional. In this sense, we follow the historical path of Einstein's derivation of the original field equations. At the moment, a version of our model with an action functional is not available, but its efficiency at the level of the field equations is truly remarkable, as we will show. In this sense, its phenomenological success may constitute a first significant step in the way of finding a solution of the difficult CC problem.

The present Letter is organized as follows. In Section 2 we present the basic setup of our model. In Section 3 we present a toy model of the CC relaxation mechanism which helps to understand the basic idea behind our proposal, although it is still too simple to describe our Universe. Only in Section 4 we present a first realistic version of the full relaxation mechanism and we perform a numerical analysis of it. In Section 5, we discuss in more detail some aspects and implications of our model. Finally, in the last section we draw our conclusions.

2. The setup

We start from the generalized Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi}{M_P^2}\left(T_{\mu\nu}^m + T_{\mu\nu}^X + g_{\mu\nu}\rho_{\Lambda,\text{eff}}\right), \tag{1}$$

where $T_{\mu\nu}^m$ is the energy–momentum tensor of ordinary matter – including the energy densities of radiation (ρ_r) and baryons (ρ_b). Furthermore, $T_{\mu\nu}^X$ describes the X component (ρ_X), interacting with the effective CC term $g_{\mu\nu}\rho_{\Lambda,\text{eff}}$ in such a way that the total density of the dark sector, $\rho_D = \rho_{\Lambda,\text{eff}} + \rho_X$, is covariantly conserved (in accordance with the Bianchi identity). The effective CC density $\rho_{\Lambda,\text{eff}}$ is given by

$$\rho_{\Lambda,\text{eff}} = \rho_\Lambda^i + \rho_{\text{inv}}. \tag{2}$$

Here, ρ_Λ^i is an arbitrarily large initial (and constant) cosmological term, and $\rho_{\text{inv}} = \rho_{\text{inv}}(R,S,T)$ is some function of the general coordinate invariant terms

$$\begin{aligned} R &\equiv R_{\mu\nu}g^{\mu\nu} = 6H^2(1-q), \\ S &\equiv R_{\mu\nu}R^{\mu\nu} = 12H^4\left[\left(\frac{1}{2}-q\right)^2 + \frac{3}{4}\right], \\ T &\equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 12H^4\left(1+q^2\right), \end{aligned} \tag{3}$$

which we have evaluated in the flat Friedmann–Robertson–Walker (FRW) metric in terms of the expansion rate $H = \dot{a}/a$, and the deceleration parameter $q = -\ddot{a}a/\dot{a}^2 = -\dot{H}/H^2 - 1$. We find it useful to write the structure of ρ_{inv} in the form

$$\rho_{\text{inv}} = \frac{\beta}{f}, \tag{4}$$

where β is a dimension 6 parameter and $f = f(R,S,T)$ is a dimension 2 function of the aforementioned invariants. This form is particularly convenient since the function f must grow at high energies and hence the vacuum energy is ultraviolet safe, i.e. in the early Universe $\rho_{\text{inv}} \to 0$ and $\rho_{\Lambda,\text{eff}} \to \rho_\Lambda^i$, where ρ_Λ^i is arbitrarily large but finite.

The generalized field equations (1) fall into the metric-based category of extensions of General Relativity. However, at this point the following observation is in order. In the literature, the extensions of Einstein's field equations are usually of a restricted class, namely those that can be derived from effective gravitational actions of the form

$$\Gamma = \int d^4x\,\sqrt{-g}\left[\frac{M_P^2}{16\pi}R + F(R,S,T)\right]. \tag{5}$$

This class of models may be called the "$F(R,S,T)$-theories" as they are characterized by an arbitrary (albeit sufficiently differentiable) local function F of the invariants defined in Eqs. (3), usually some polynomial of these invariants. Work along these lines has been put forward e.g. in [16]. The particular subclass of models in which the function F depends only on R, or "$F(R)$-theories", is well known and has been subject of major interest [17,18].

However, as advertised in the introduction, in this work we formulate the relaxation mechanism directly in terms of the generalized field equations (1), without investigating at the moment the eventual connection with an appropriate effective action. The reason is, basically, because we aim at maximal simplicity at the moment. To be sure, after many years of unsuccessful attempts, the CC problem has revealed itself as one of the most difficult problems (if not the most difficult one) of all theoretical physics; and we should not naively expect to shoot squarely at it and hope to hit the jackpot at the first trial, so to speak. In this sense, if we can find a way to solve, or at least to significantly improve, the problem directly at the level of the field equations, we might then find ourselves in a truly vantage point to subsequently attempt solving the CC problem at the level of some generalized form of the effective action of gravity.

All in all, let us warn the reader that the connection between the two approaches (viz. the one based on the field equations and the functional one) is, if existent, non-trivial. In fact, we note that the presumed action behind the field equations (1) need not be of the local form (5), and in general we cannot exclude that it may involve some complicated contribution from non-local terms. These terms, however involved they might be, are nevertheless welcome and have been advocated in the recent literature as a possible solution to the dark energy problem from different perspectives [19]. In the present work, we wish to put aside the discussion of these terms, and, for that matter, all issues related to the hypothetical action functional behind our field equations. Instead, we want to exclusively concentrate on the phenomenological possibilities that our framework can provide on relaxing the effective CC term (2) depending on the choice of the function f. In particular, if the