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Magnetic string coupled to nonlinear electromagnetic field

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ABSTRACT

We introduce a class of rotating magnetically charged string solutions of the Einstein gravity with a nonlinear electrodynamics source in four dimensions. The present solutions has no curvature singularity and no horizons but has a conic singularity and yields a spacetime with a longitudinal magnetic field. Also, we investigate the effects of nonlinearity on the properties of the solutions and find that for the special range of the nonlinear parameter, the solutions are not asymptotic AdS. We show that when the rotation parameter is nonzero, the spinning string has a net electric charge that is proportional to the magnitude of the rotation parameter. Finally, we use the counterterm method inspired by AdS/CFT correspondence and calculate the conserved quantities of the solutions.

1. Introduction

Topological defects are inevitably formed during phase transitions in the early universe, and their subsequent evolution and observational signatures must therefore be understood. The string model of structure formation may help to resolve one of cosmological mystery, the origin of cosmic magnetic fields [1]. There is strong evidence from all numerical simulations for the scaling behavior of the long string network during the radiation-dominated era. Apart from their possible astrophysical roles, topological defects are fascinating objects in their own right. Their properties, which are very different from those of more familiar system, can give rise to a rich variety of unusual mathematical and physical phenomena [2].

On another front, nonlinear electromagnetic fields are subjects of interest for a long time. For example, there has been a renewed interest in Born–Infeld gravity ever since new solutions have been found in the low energy limit of string theory. Static and rotating solutions of Born–Infeld gravity have been considered in Refs. [3–5].

In this Letter, we turn to the investigation of spacetimes generated by static and spinning string sources in four-dimensional Einstein theory in the presence of a nonlinear electromagnetic field which are horizonless and have nontrivial external solutions. The basic motivation for studying these kinds of solutions is that they may be interpreted as cosmic strings. Cosmic strings are topological structure that arise from the possible phase transitions to which the universe might have been subjected to and may play an important role in the formation of primordial structures. A short review of papers treating this subject follows. Solutions of Einstein's equations with conical singularities describing straight strings can easily be constructed [6]. One needs only a spacetime with a symmetry axis. If one then cuts out a wedge then a space with a string lying along the axis is obtained. A nonaxisymmetric solutions of the combined Einstein and Maxwell equations with a string has been found by Linet [7]. The four-dimensional horizonless solutions of Einstein gravity have been explored in [8,9]. These horizonless solutions [8,9] have a conical geometry; they are everywhere flat except at the location of the line source. The spacetime can be obtained from the flat spacetime by cutting out a wedge and identifying its edges. The wedge has an opening angle which turns to be proportional to the source mass. The extension to include the Maxwell field has also been done [10]. Static and spinning magnetic sources in three and four-dimensional Einstein–Maxwell gravity with negative cosmological constant have been explored in [11,12]. The generalization of these asymptotically AdS magnetic rotating solutions to higher dimensions has also been done [13]. In the context of electromagnetic cosmic string, it has been shown that there are cosmic strings, known as superconducting cosmic strings, that behave as superconductors and have interesting interactions with astrophysical magnetic fields [14]. The properties of these superconducting cosmic strings have been investigated in [15]. Solutions with longitudinal and angular magnetic field were considered in Refs. [16–19]. Similar static solutions in the context of cosmic string theory were found in Ref. [20]. All

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of these solutions [16–18,20,21] are horizonless and have a conical geometry; they are everywhere flat except at the location of the line source. The extension to include the electromagnetic field has also been done [22,23]. The generalization of these solutions in Einstein gravity in the presence of a dilaton and Born–Infeld electromagnetic fields has been done in Ref. [24].

Another example of the nonlinear electromagnetic field is conformally invariant Maxwell field. In many papers, straightforward generalization of the Maxwell field to higher dimensions one essential property of the electromagnetic field is lost, namely, conformal invariance. Indeed, in the Reissner–Nordström solution, the source is given by the Maxwell action which enjoys the conformal invariance in four dimensions. Massless spin-1/2 fields have vanishing classical stress tensor trace in any dimension, while scalars can be "improved" to achieve $T^{\alpha}_{\alpha}=0$, thereby guaranteeing invariance under the special conformal (or full Weyl) group, in accord with their scale-independence [25]. Maxwell theory can be studied in a gauge which is invariant under conformal rescalings of the metric, and at first, have been proposed by Eastwood and Singer [26]. Also, Poplawski [27] have been showed the equivalence between the Ferraris–Kijowski and Maxwell Lagrangian results from the invariance of the latter under conformal transformations of the metric tensor. Quantized Maxwell theory in a conformally invariant gauge have been investigated by Esposito [28]. In recent years, gravity in the presence of nonlinear and conformally invariant Maxwell source have been studied in many papers [29,30].

The outline of our Letter is as follows. We give a brief review of the field equations of Einstein gravity in the presence of cosmological constant and nonlinear electromagnetic field in Section 2. In Section 3 we present static horizonless solutions which produce longitudinal magnetic field, compare it with the solutions of the standard electromagnetic field and then investigate the properties of the solutions and the effects of nonlinearity of the electromagnetic field on the deficit angle of the spacetime. Section 4 will be devoted to the generalization of these solutions to the case of rotating solutions and use of the counterterm method to compute the conserved quantities of them. We finish our Letter with some concluding remarks.

2. Basic field equations

Our starting point is the four-dimensional Einstein-nonlinear Maxwell action

$$I_G=-\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R-2\Lambda-\alpha F^s\right)-\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3x\sqrt{-\gamma}\,\Theta(\gamma), \tag{1}$$

where R is the scalar curvature, Λ is the cosmological constant, F is the Maxwell invariant which is equal to $F_{\mu\nu}F^{\mu\nu}$ (where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic tensor field and A_μ is the vector potential), α and s is a coupling and arbitrary constant respectively. The last term in Eq. (1) is the Gibbons–Hawking surface term. It is required for the variational principle to be well-defined. The factor Θ represents the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and γ is the induced metric on the boundary. Varying the action (1) with respect to the gravitational field $g_{\mu\nu}$ and the gauge field A_μ, yields

$$G_{\mu\nu}+\Lambda g_{\mu\nu}=T_{\mu\nu}, \tag{2}$$

$$\partial_\mu\left(\sqrt{-g}F^{\mu\nu}F^{s-1}\right)=0. \tag{3}$$

In the presence of nonlinear electrodynamics field, the energy–momentum tensor of Eq. (2) is

$$T_{\mu\nu}=2\alpha\left[sF_{\mu\rho}F^{\rho}_{\nu}F^{s-1}-\frac{1}{4}g_{\mu\nu}F^s\right]. \tag{4}$$

The conserved mass and angular momentum of the solutions of the above field equations can be calculated through the use of the substraction method of Brown and York [31]. Such a procedure causes the resulting physical quantities to depend on the choice of reference background. A well-known method of dealing with this divergence for asymptotically AdS solutions of Einstein gravity is through the use of counterterm method inspired by AdS/CFT correspondence [32]. In this Letter, we deal with the spacetimes with zero curvature boundary, $R_{abcd}(\gamma)=0$, and therefore the counterterm for the stress–energy tensor should be proportional to γ^{ab}. We find the suitable counterterm which removes the divergences as

$$I_{ct}=-\frac{1}{4\pi}\int_{\partial\mathcal{M}} d^3x\frac{\sqrt{-\gamma}}{l}. \tag{5}$$

Having the total finite action $I=I_G+I_{\mathrm{ct}}$, one can use the quasilocal definition to construct a divergence free stress–energy tensor [31]. Thus the finite stress–energy tensor in four-dimensional Einstein-nonlinear Maxwell gravity with negative cosmological constant can be written as

$$T^{ab}=\frac{1}{8\pi}\left[\Theta^{ab}-\Theta\gamma^{ab}+\frac{2\gamma^{ab}}{l}\right]. \tag{6}$$

The first two terms in Eq. (6) are the variation of the action (1) with respect to γ_{ab}, and the last term is the variation of the boundary counterterm (5) with respect to γ_{ab}. To compute the conserved charges of the spacetime, one should choose a spacelike surface $\mathcal{B}$ in $\partial\mathcal{M}$ with metric σ_{ij}, and write the boundary metric in ADM (Arnowitt–Deser–Misner) form:

$$\gamma_{ab}\,dx^a\,dx^a=-N^2\,dt^2+\sigma_{ij}\left(d\varphi^i+V^i\,dt\right)\left(d\varphi^j+V^j\,dt\right),$$

where the coordinates φ^i are the angular variables parameterizing the hypersurface of constant r around the origin, and N and V^i are the lapse and shift functions, respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (6) can be written as

$$Q(\xi)=\int_{\mathcal{B}} d^2x\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{7}$$

where σ is the determinant of the metric σ_{ij}, ξ and n^a are, respectively, the Killing vector field and the unit normal vector on the boundary $\mathcal{B}$. For boundaries with timelike ($\xi=\partial/\partial t$) and rotational ($\varsigma=\partial/\partial\phi$) Killing vector fields, one obtains the quasilocal mass and angular momentum

$$M=\int_{\mathcal{B}} d^2x\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{8}$$

$$J=\int_{\mathcal{B}} d^2x\sqrt{\sigma}T_{ab}n^a\varsigma^b. \tag{9}$$

These quantities are, respectively, the conserved mass and angular momentum of the system enclosed by the boundary $\mathcal{B}$. Note that they will both depend on the location of the boundary $\mathcal{B}$ in the spacetime, although each is independent of the particular choice of foliation $\mathcal{B}$ within the surface $\partial\mathcal{M}$.