Physics Letters B 697 (2011) 70–74

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

A two-component picture of the $\langle A_\mu^2 \rangle$ condensate with instantons

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ARTICLE INFO

Article history:
Received 23 September 2010
Received in revised form 14 January 2011
Accepted 17 January 2011
Available online 20 January 2011
Editor: B. Grinstein

Keywords:
Yang–Mills vacuum
Condensates
Instanton

ABSTRACT

We study the interplay between the $\langle A_\mu^2 \rangle$ condensate and instantons in non-Abelian gauge theory. Therefore we use the formalism of Local Composite Operators, with which the vacuum expectation value of this condensate can be analytically computed. We first use the dilute gas approximation and partially solve the infrared problem of instanton physics. In order to find quantitative results, however, we turn to an instanton liquid model, where we find how the different contributions to the condensate add up.

1. Introduction

The dimension 2 gluon condensate $\langle A_\mu^2 \rangle$ in pure Yang–Mills theory has been proposed in [1,2], and it has been investigated in different ways since then [3–14].

In [3] an analytical framework for studying this condensate has been developed, based on work carried out in the Gross–Neveu model [15]. Different problems had to be overcome. First of all there is the gauge invariance of this condensate. In order to make the operator A_μ^2 gauge invariant, one can take the minimum of its integral over the gauge orbit. Since $\int d^d x\, A_\mu^U A_\mu^U$, with $U \in SU(N)$, is positive, this minimum will always exist. In a general gauge, however, the minimum is a highly non-local and thus hard to handle expression of the gauge field. A minimum is however reached in the Landau gauge ($\partial_\mu A_\mu = 0$), such that working in this gauge reduces the operator to a local expression.[1] Secondly adding a source J, coupled to A_μ^2, makes the theory non-renormalizable at the quantum level. To solve this, a term quadratic in the source must be added, which in turn spoils the energy interpretation of the effective action. One way around this is to perform the Legendre inversion, but this is rather cumbersome, especially with a general, space–time dependent source. One can also use a Hubbard–Stratonovich transform, which introduces an auxiliary field (whose interpretation is just the condensate) and eliminates the term quadratic in the source. Details can be found in [3]. The result was that the Yang–Mills vacuum favors a finite value for the expectation value of A_μ^2. The precise renormalization details of the procedure proposed in [3] were given in [4]. We review this formalism in Section 2.

Instantons play an important role in the QCD vacuum and have a large influence in many infrared properties (see [16] for a review). As such it is an interesting question what their connection with the dimension two condensate is. A first study in this direction has been done on the lattice by Boucaud et al. [17,18], and a rather large instanton contribution to the condensate has been found, which shows some agreement with the results from an OPE approach to the gluon propagator from [8]. However, the condensate may get separate contributions from other sources, as for example the non-perturbative high-energy fluctuations leading to the condensate found in [3]. The opposite viewpoint is just as interesting: what is the influence of an effective gluon mass on the instanton ensemble? In 't Hooft's seminal paper he found that, in a Higgs model, a gauge boson mass stabilizes the instanton gas [19].

Some subtle points are to be resolved before a full treatment can be given. These are discussed in Section 3. Then, Section 4 is devoted to the computation of the one-loop effective action, for which we use the strategy developed by Dunne et al. [20,21]. Finally, Section 5 concludes this Letter.

2. $\langle A_\mu^2 \rangle$ and instantons

In this section we will review the LCO formalism as proposed in [3] and modified to use it with a background field.

As a first step the gauge is fixed using the Landau condition, i.e. the linear covariant gauge $\partial_\mu A_\mu = 0$ with $\xi \to 0$. Then, a term

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[1] We ignore the Gribov problem here, see also [7].

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2011.01.029

$$\frac{1}{2}JA_\mu^2 \tag{1}$$

is added to the Lagrangian density. Here J is the source which will be used to compute $\langle A_\mu^2\rangle$. As it stands, the theory is not renormalizable. To correct this, a new term

$$-\frac{1}{2}\zeta J^2 \tag{2}$$

has to be added. Here ζ is a new coupling constant which will have to be determined as a function of the parameters in the original theory. This Lagrangian is now multiplicatively renormalizable, as shown in [22] using a BRST analysis.

As we want to work with an instanton as a background field, it is more appropriate to use the Landau background gauge [23] $\mathcal{D}_\mu[\hat{A}]A_\mu = 0$ instead of the usual Landau gauge prescription $\partial_\mu A_\mu = 0$. Here $\hat{A}_\mu$ is the background field. In order to do so, some alterations are in order. A BRST analysis (for BRST in the background gauge, see for example [24]) shows that, in order for the LCO formalism to stay renormalizable, the condensate A_μ^2 must be replaced by

$$(A_\mu - \hat{A}_\mu)^2 = \mathcal{A}_\mu^2 \tag{3}$$

with A_μ the total gauge field and $\mathcal{A}_\mu$ the quantum fluctuations, $A_\mu = \mathcal{A}_\mu + \hat{A}_\mu$.

In order for this formalism to work, some creases have to be ironed out. As a first point, we have introduced a new parameter, ζ, creating a problem of uniqueness. However, it is possible to choose ζ to be a unique meromorphic function of g^2 based on the renormalization group equations. In [3] there was found, using the $\Lambda_{\overline{\mathrm{MS}}}$ scheme in $d = 4-\epsilon$ dimensions and without any background field (up to one-loop order and with N_c the number of colors):

$$\zeta = \frac{9}{13}\frac{N_c^2-1}{N_c}\frac{1}{g^2} + \frac{N_c^2-1}{16\pi^2}\frac{161}{52} + \mathcal{O}(g^2), \tag{4a}$$

$$Z_\zeta = 1 - \frac{g^2 N_c}{16\pi^2}\frac{13}{3\epsilon} + \mathcal{O}(g^2), \tag{4b}$$

$$Z_2 = 1 - \frac{N_c g^2}{16\pi^2}\frac{3}{2\epsilon} + \mathcal{O}(g^2), \tag{4c}$$

where Z_ζ and Z_2 are the constants renormalizing ζJ^2 and $J\mathcal{A}_\mu^2$ respectively. For dimensional reasons, working in the background gauge will change nothing to the expressions for ζ and the renormalization constants.

Secondly the presence of the J^2 term spoils an energy interpretation for the effective potential. One way around this is to perform the Legendre inversion, but this is rather cumbersome, especially so with a general, space–time dependent source. A more elegant way out applies a Hubbard–Stratonovich transformation by inserting unity into the path integral:

$$1 = \mathcal{N}\int[\mathcal{D}\sigma]\exp -\frac{1}{2\zeta}\int\left(\frac{\sigma}{g} + \frac{1}{2}\mathcal{A}_\mu^2 - \zeta J\right)^2 d^4x \tag{5}$$

with $\mathcal{N}$ an irrelevant constant. This eliminates the $\frac{1}{2}J\mathcal{A}_\mu^2$ and ζJ^2 terms from the Lagrangian and introduces a new field σ. The result is:

$$\begin{aligned} e^{-W(J)} = \int[\mathcal{D}\mathcal{A}_\mu][\mathcal{D}\sigma]\exp \\ -\int\bigg(\mathcal{L}_{\mathrm{YM}}[\mathcal{A}_\mu,\hat{A}_\mu,c,\bar{c}] \\ + \mathcal{L}_{\mathrm{LCO}}[\mathcal{A}_\mu,\hat{A}_\mu,\sigma] - \frac{\sigma}{g}J\bigg)d^4x. \end{aligned} \tag{6}$$

Here $\mathcal{L}_{\mathrm{YM}}$ is the well-known Yang–Mills Lagrangian with Faddeev–Popov ghosts, fixed in the Landau background gauge, and

$$\mathcal{L}_{\mathrm{LCO}}[\mathcal{A}_\mu,\sigma] = \frac{\sigma^2}{2g^2\zeta} + \frac{\sigma\mathcal{A}_\mu^2}{2g\zeta} + \frac{(\mathcal{A}_\mu^2)^2}{8\zeta}. \tag{7}$$

Now J acts as a linear source for the σ field, so that we can straightforwardly compute the effective action $\Gamma(\sigma)$ using the above expressions.

If we compare our new Lagrangian to the original expression, we find that the expectation value of σ corresponds to the expectation value of the composite operator

$$\langle\sigma\rangle = -g\left\langle\frac{1}{2}\mathcal{A}_\mu^2 - \zeta J\right\rangle. \tag{8}$$

In the limit $J \to 0$ this operator corresponds (up to a multiplicative factor) to $\mathcal{A}_\mu^2$. We can also read off the effective gluon mass in the lowest order:

$$m^2 = \frac{\sigma}{g\zeta} = \frac{N_c}{N_c^2-1}\frac{13}{9}g\sigma. \tag{9}$$

3. Instantons and $\langle A_\mu^2\rangle$

Let us first look into whether the condensate $\langle A_\mu^2\rangle$ can stabilize the instanton ensemble in the LCO formalism, as, if successful, it would minimize the amount of hand-waving necessary to compute the action. First we have the question of which gauge to choose. All instanton calculations are done in background gauges, as analytic computations in non-background gauges are quite impossible. The LCO formalism does not give classical fields a mass in the Landau background gauge, however. In the electroweak theory considered by 't Hooft in [19] it is exactly this classical mass which suppresses large instantons by the simple fact that large instantons are no solutions to the massive field equations anymore, while small instantons can still be considered approximate solutions.

If we want to have a mass already at the classical level, it is necessary to work in the non-background Landau gauge. Although the computations cannot be carried through in this gauge, it still possible to find the qualitative form of the result. In order to circumvent the question of which background to take for the σ field[2] it is more opportune to start before the point where the Hubbard–Stratonovich transformation is introduced.

We start from

$$-\frac{1}{2}\langle A_\mu^2\rangle = \frac{\delta}{\delta J}\ln\int[dA_\mu]e^{-S-\frac{1}{2}JA_\mu^2+\frac{\zeta}{2}J^2}\bigg|_{J=0}. \tag{10}$$

As the source is small, instantons will be approximate solutions. Eventually, we can correct the instanton using the valley method [25], but this turns out not to give more insight. At the classical level, the action of the instanton is now

$$S + \frac{1}{2}JA_\mu^2 = \frac{8\pi^2}{g^2} + \frac{6\pi^2}{g^2}J\rho^2 + \cdots, \tag{11}$$

where the dots stand for contributions from corrections to the instanton solution. From renormalization group arguments, we can now write down the general form of the one-loop result:

$$W[J] = W^{0I}[J] - \int_0^\infty \frac{d\rho}{\rho^5}\exp\bigg(-\frac{8\pi^2}{g^2} - \frac{6\pi^2}{g^2}J\rho^2$$

[2] Allowing σ to obey its own *classical* field equations does not lead to non-trivial results.