Physics Letters B 693 (2010) 555–558

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

QED is not endangered by the proton's size

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ARTICLE INFO

Article history:
Received 30 August 2010
Accepted 31 August 2010
Available online 9 September 2010
Editor: L. Alvarez-Gaumé

Keywords:
Proton radius
Lyman shift
Muonic hydrogen

ABSTRACT

Pohl et al. have reported a very precise measurement of the Lamb-shift in muonic hydrogen (Pohl et al., 2010) [1], from which they infer the radius characterizing the proton's charge distribution. The result is 5 standard deviations away from the one of the CODATA compilation of physical constants. This has been interpreted (Pohl et al., 2010) [1] as possibly requiring a 4.9 standard-deviation modification of the Rydberg constant, to a new value that would be precise to 3.3 parts in 10^{13}, as well as putative evidence for physics beyond the standard model (Flowers, 2010) [2]. I demonstrate that these options are unsubstantiated.

1. Introduction

The issue is extremely simple. The discrepancy quoted in the abstract is between results which do not depend on a specific model of the proton's form factor and results, by Pohl et al., which do [3]. The conclusion is not that the experiments or the theory are wrong, but that the model (the customary dipole form factor) is inadequate at the level of precision demanded by the data. The experiments and QED are right, the dipole is wrong. More generally, it is risky to use a model of the charge distribution to extract a property of the very same charge distribution.

The conclusion of the previous paragraph is the expected one. The dipole form factor is but a rough description of higher-energy data and is unacceptable on grounds of the analyticity requirements stemming from causality and the locality of fundamental interactions.

Moreover, any simple one-parameter description of the proton's non-relativistic Sacks form factor, $G_E(-\mathbf{q}^2)$ in terms of only one mass parameter is inaccurate: the proton is not so simple. More precisely, the proton's relativistic form factor, $G_E(q^2)$, is expected, in the timelike domain $q^2 \geqslant 0$, to have a complex structure, with a first cut starting at $q^2 = 4m_\pi^2$ and a plethora of branch cuts and complex resonant poles thereafter [4].

The same is true of the charge distribution, $\rho_p(r)$, the Fourier transform of $G_E(-\mathbf{q}^2)$. Even most naively, $\rho_p(r)$ is expected to have a "core" and a "pion cloud" [5], corresponding to a minimum of two length parameters.

2. In detail

Let ℓ denote an electron or a μ^-. The leading proton-size correction to the energy levels of an ℓp atom is

$$\Delta E = \frac{2\alpha^4}{3n^3} m_r^3 \delta_{l0} \langle r_p^2 \rangle,$$
$$m_r \equiv \frac{m_\ell m_p}{m_\ell + m_p} \tag{1}$$

where $\langle r_p^2 \rangle$ is the mean square radius of $\rho_p(r)$.

The charge distribution is related to the non-relativistic limit of the electric form-factor, G_E, by the Fourier transformation

$$G_E(-\mathbf{q}^2) = \int d^3r\, \rho_p(\mathbf{r}) e^{-i\vec{q}\vec{r}}. \tag{2}$$

Precise measurements of $\langle r_p^2 \rangle$ have two origins. One is mainly based on the theory [6] and observations [7] of hydrogen. The result, compiled in CODATA [8], is

$$\langle r_p^2 \rangle(\text{CODATA}) = (0.8768 \pm 0.0069\ \text{f})^2. \tag{3}$$

The second type of measurement is based on the theory and observations [9,10] of very low-energy electron–proton scattering. It yields

$$\langle r_p^2 \rangle(ep) = (0.895 \pm 0.018\ \text{f})^2. \tag{4}$$

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.08.074

This result requires a sophisticated data analysis, partly based on a continued-fraction expansion of G_E [9].

The two quoted methods of measuring $\langle r_p^2 \rangle$ are model-independent, in the sense of not assuming a particular form of the proton's charge distribution, $\rho_p(r)$.

The plot thickens as one considers the Lamb shift $2P_{3/2}^{F=2} \to 2S_{1/2}^{F=1}$ in the μp atom, measured [1] to be

$$L_{\mathrm{exp}} = 206.2949 \pm 0.0032 \text{ meV}. \tag{5}$$

In meV units for energy and fermi units for the radii, the predicted value [11] is of the form

$$L^{\mathrm{th}}\big[\langle r_p^2 \rangle, \langle r_p^3 \rangle_{(2)}\big] = 209.9779(49) - 5.2262\langle r_p^2 \rangle + 0.00913\langle r_p^3 \rangle_{(2)}. \tag{6}$$

The first two coefficients are best estimates of many contributions while the third stems from the $n = 2$ value of an addend [12,6]

$$\Delta E_3(n) = \frac{\alpha^5}{3n^3} m_r^4 \delta_{l0} \langle r_p^3 \rangle_{(2)}, \tag{7}$$

proportional to the third Zemach moment

$$\langle r_p^3 \rangle_{(2)} \equiv \int d^3 r_1\, d^3 r_2\, \rho(r_1)\rho(r_2)|\mathbf{r}_1 - \mathbf{r}_2|^3. \tag{8}$$

For a single-parameter description of the charge distribution, there is an explicit relation between $\langle r_p^3 \rangle_{(2)}$ and $\langle r_p^2 \rangle$. Consider, as an example, a ρ-dominated form factor in its narrow-width non-relativistic limit

$$G_E(q^2) = \frac{m_\rho^2}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho} \to \frac{m_\rho^2}{\mathbf{q}^2 + m_\rho^2}. \tag{9}$$

The corresponding charge distribution is a Yukawian

$$\rho(r) = \frac{m_\rho^2}{4\pi r} e^{-m_\rho r}. \tag{10}$$

Its relevant moments are $\langle r^0 \rangle = 1$,

$$\langle r^2 \rangle = 6/m_\rho^2, \qquad \langle r^3 \rangle = 24/m_\rho^3, \qquad \langle r^3 \rangle_{(2)} = 60/m_\rho^3. \tag{11}$$

The model-dependent relation is thus

$$\big[\langle r^3 \rangle_{(2)}\big]^2 = \frac{50}{3}\big[\langle r^2 \rangle\big]^3. \tag{12}$$

For a dipole form factor

$$G_E(-\mathbf{q}^2) = \frac{m_d^4}{(\mathbf{q}^2 + m_d^2)^2} \tag{13}$$

the charge distribution is an exponential

$$\rho(r) = \frac{m_d^3}{8\pi} e^{-m_d r} \tag{14}$$

for which $\langle r^0 \rangle = 1$,

$$\langle r^2 \rangle = 12/m_d^2, \qquad \langle r^3 \rangle = 60/m_d^3, \qquad \langle r^3 \rangle_{(2)} = 315/(2m_d^3). \tag{15}$$

The model-dependent relation is thus

$$\big[\langle r^3 \rangle_{(2)}\big]^2 = \frac{3675}{64}\big[\langle r^2 \rangle\big]^3. \tag{16}$$

The ratio of the numbers in Eqs. (12), (16) is $128/441 \sim 0.29$, showing the difference of relevant moments between to form-factor "models". Even if we took the sixth root of this number to bring it closer to unity – as experimentalists do with $\langle r^2 \rangle$ to halve the relative error – the result would, at the required great precision, still epitomize the model-dependence of the results.

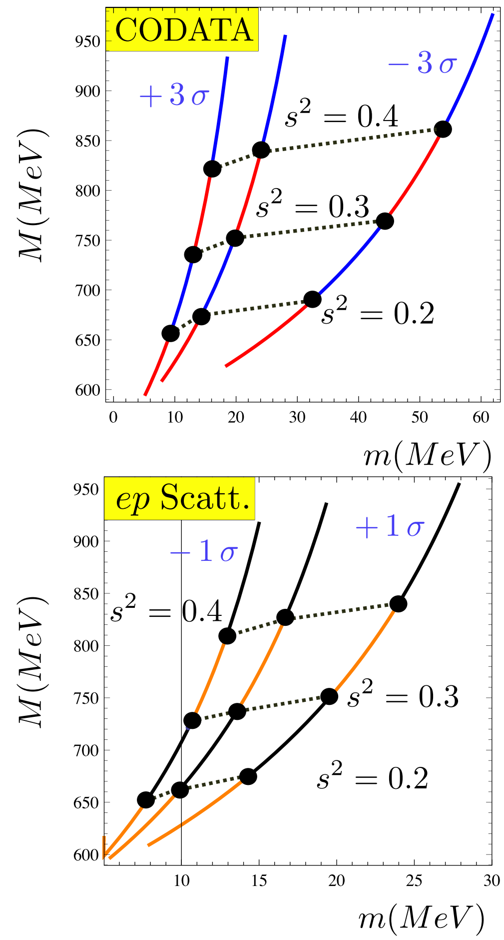

Fig. 1. Parameters M and m for which the toy model is compatible with the data, with $s^2 = \sin^2(\theta)$ varying along the curves, see Eqs. (20), (21). Top: Lyman in the μp atom and CODATA, shown for the central value and a very asymmetric $\pm 3\sigma$. Bottom: CODATA substituted for ep scattering, central value and $\pm 1\sigma$ (there is no solution for $+3\sigma$).

2.1. A toy model

The photon propagator in the time-like domain ($q^2 > 0$) has led to considerable revolutions (e.g. the discovery and interpretation of the J/Ψ), as well as interesting challenges, in particular close to its cut at $q^2 \geqslant 4m_\pi^2$. The modeling of the electric and magnetic form factors G_E and G_M of protons and neutrons in terms of dispersion relations for the photon propagator involves, literally, dozens of parameters [4]. The form-factor "toy model" I am going to discuss is not intended to compete in accuracy with the dispersive approaches, nor to be a realistic description of ep data, but only to elucidate the current discussion.

In [4], an accurate description of the theoretically-calculated 2π continuum required products of up to three poles. I parametrize $\rho(r)$ as an interpolation between the charge densities of a "ρ" single pole and a "2π" dipole:

$$\rho(r) = \frac{1}{D}\left[\frac{M^4 e^{-Mr}\cos^2(\theta)}{4\pi r} + \frac{m^5 e^{-mr}\sin^2(\theta)}{8\pi}\right],$$
$$D \equiv M^2\cos^2(\theta) + m^2\sin^2(\theta) \tag{17}$$

whose two first relevant moments are $\langle r^0 \rangle = 1$ and

$$\langle r^2 \rangle\big|_{\mathrm{toy}} = \frac{6}{m^2\tan^2(\theta) + M^2} + \frac{12}{m^2 + M^2\cot^2(\theta)}. \tag{18}$$