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Gluon condensates and c, b quark masses from quarkonia ratios of moments

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ABSTRACT

We extract (for the first time) the ratio of the gluon condensate $\langle g^3 f_{abc} G^3\rangle/\langle\alpha_s G^2\rangle$ expressed in terms of the liquid instanton radius ρ_c from charmonium moments sum rules by examining the effects of $\langle\alpha_s G^2\rangle$ in the determinations of both ρ_c and the running $\overline{\mathrm{MS}}$ mass $\overline{m}_c(m_c)$. Using a global analysis of selected ratios of moments at different $Q^2 = 0$, $4m_c^2$ and $8m_c^2$ and keeping $\langle\alpha_s G^2\rangle$ from 0.06 GeV^4, where the estimate of ρ_c is almost independent of $\langle\alpha_s G^2\rangle$, we deduce: $\rho_c = 0.98(21)$ GeV^{-1} corresponding to $\langle g^3 f_{abc} G^3\rangle = (31 \pm 13)$ GeV$^2\langle\alpha_s G^2\rangle$. The value of $\overline{m}_c(m_c)$ is less affected (within the errors) by the variation of $\langle\alpha_s G^2\rangle$, where a common solution from different moments are reached for $\langle\alpha_s G^2\rangle \geqslant 0.02$ GeV^4. Using the values of $\langle\alpha_s G^2\rangle = 0.06(2)$ GeV^4 from some other channels and the previous value of $\langle g^3 f_{abc} G^3\rangle$, we deduce: $\overline{m}_c(m_c) = 1261(18)$ MeV and $\overline{m}_b(m_b) = 4232(10)$ MeV, where an estimate of the 4-loops ($\mathcal{O}(\alpha_s^3)$) contribution has been included. Our analysis indicates that the errors in the determinations of the charm quark mass and of α_s without taking into account the ones of the gluon condensates have been underestimated. To that accuracy, one can deduce the running light and heavy quark masses and their ratios evaluated at M_Z, where it is remarkable to notice the approximate equalities: $m_s/m_u \approx m_b/m_s \approx m_t/m_b \approx 51(9)$, which might reveal some eventual underlying novel symmetry of the quark mass matrix in some Grand Unified Theories.

1. Introduction

Non-zero values of the gluon condensates have been advocated by SVZ [1,2]. Indeed, the gluon condensates play an important role in gluodynamics (low-energy theorems, ...) and in some bag models as they are directly related to the vacuum energy density (with standard notations):

$$E = -\frac{\beta(\alpha_s)}{8\alpha_s^2}\langle\alpha_s G^2\rangle. \tag{1}$$

Moreover, the gluon condensates enter in the OPE of the hadronic correlators [1] and then are important in the analysis of QCD spectral sum rules (QSSR), especially, in the heavy quarks and in the pure Yang–Mills gluonia channels where the light quark loops and quark condensates[1] are absent to leading order [3–5]. The SVZ value:

$$\langle\alpha_s G^2\rangle \simeq 0.04\ \mathrm{GeV}^4, \tag{2}$$

extracted (for the first time) from charmonium sum rules [1] has been challenged by different authors [3–5]. Though there are strong indications that the exact value of the gluon condensate is around this value or most likely 2 times this value as obtained from heavy quarks exponential moments [3–6], heavy quark mass-splittings [7] and e^+e^- [8–10], most recent determinations from τ-decay [11–13] (see however [14]) and the previous charmonium moments [15] indicate that its value is not well determined. In fact, at present, the structure of the QCD vacuum is not yet under a good control. If one follows the SVZ idea based on the ordinary OPE, the QCD confinement can be parametrized by the sum of quark and gluon condensates of higher and higher dimensions.[2] In order to estimate the higher dimension condensates, one usually assumes factorization using vacuum saturation (leading $1/N_c$ approximation). However, in many examples, this assumption appears to be badly violated [8,9,11–13,18–24]. Different phenomenological works have been performed for understanding the complex structure of the QCD vacuum in the $V + A$ and $V - A$ channels of the light flavours [8,9,11–13,18–24] and from lattice calculations [25,27,28]. Here, we shall estimate (for the first time) the ratio of the dimension-6 $\langle g^3 f_{abc} G^3\rangle$ over the dimension-4 $\langle\alpha_s G^2\rangle$ gluon condensates using charmonium sum rules,[3] in the

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[1] The heavy quark condensate contribution can be absorbed into the gluon one through the relation [1]: $\langle\bar{Q}Q\rangle = -\langle\alpha_s G^2\rangle/(12\pi m_Q) + \cdots$. An analogous relation also occurs for the mixed quark–gluon condensate [3–5].

[2] A possible existence of an additional $1/Q^2$ term induced by large order terms of PT series has been discussed in [16,17].

[3] The $\langle g^3 f_{abc} G^3\rangle$ condensate does not contribute in the chiral limit $m_q = 0$ in the vector and axial-vector channels of light flavours.

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Table 1
Masses and electronic widths of the J/ψ family from PDG 08 [31].

Name	Mass [MeV]	$\Gamma_{J/\psi\to e^+e^-}$ [keV]
$J/\psi(1S)$	3096.916(11)	5.55(14)
$\psi(2S)$	3686.093(34)	2.33(7)
$\psi(3770)$	3775.2(1.7)	0.259(16)
$\psi(4040)$	4039(1)	0.86(7)
$\psi(4160)$	4153(3)	0.83(7)
$\psi(4415)$	4421(4)	0.58(7)

aim to clarify the different inaccurate proposals from some instanton liquid models. In so doing, we find that it is convenient to introduce the instanton radius ρ_c:

$$\frac{\langle g^3 f_{abc}G^3\rangle}{\langle\alpha_s G^2\rangle}=\frac{4}{5}\frac{12\pi}{\rho_c^2}. \tag{3}$$

The value of ρ_c ranges from $1/3$ fm $=1.5$ GeV^{-1} [29], 0.5 fm $=2.5$ GeV^{-1} [30] to 0.9 fm $=4.5$ GeV^{-1} [1]. As $\langle g^3 f_{abc}G^3\rangle$ contributes like $1/\rho_c^2$ in the OPE analysis, a more precise value of ρ_c is crucial for checking the convergence of the OPE. The estimate of ρ_c from charmonium sum rules is feasible as the light quark condensates $m_q\langle\bar qq\rangle$ contributes, to higher loop order and which are chiral suppressed are negligible, while the heavy quark condensate contribution can be absorbed into the gluon one as mentioned earlier.

2. Moment sum rules

Here, we shall be concerned with the two-point correlator of a heavy quark Q:

$$-\left(g^{\mu\nu}q^2-q^\mu q^\nu\right)\Pi_Q\left(q^2\right)\equiv i\int d^4x\, e^{-iqx}\langle 0|\mathcal{T} J_Q^\mu(x)\left(J_Q^\nu(0)\right)^\dagger|0\rangle, \tag{4}$$

where $J_Q^\mu=\bar Q\gamma^\mu Q$ is the heavy quark neutral vector current. $\mathrm{Im}\,\Pi_c(s)$ can be related to the charmonium leptonic widths and masses. In a narrow width approximation (NWA):

$$\mathcal{R}_c(t)\equiv 4\pi\,\mathrm{Im}\,\Pi_c(t+i\epsilon)=\frac{N_c}{Q_c^2\alpha^2}\sum M_\psi\Gamma_{\psi\to e^+e^-}\delta\left(t-M_\psi^2\right), \tag{5}$$

where $N_c=3$; M_ψ and $\Gamma_{\psi\to e^+e^-}$ are the mass and leptonic width of the J/ψ mesons; $Q_c=2/3$ is the charm electric charge in units of e; $\alpha=1/133.6$ is the running electromagnetic coupling evaluated at M_ψ^2. We shall use the experimental values of the J/ψ parameters compiled in Table 1.

Different forms of QSSR exist in the literature [3–5]. We shall work here with the moments:

$$\mathcal{M}_n\left(-q^2\equiv Q^2\right)=\int_{4m_Q^2}^{\infty} dt\,\frac{\mathcal{R}_c(t,m_c^2)}{(t+Q^2)^{n+1}}, \tag{6}$$

and more likely with their ratios:

$$r_{n/n+1}\left(Q^2\right)=\frac{\mathcal{M}_n}{\mathcal{M}_{n+1}},\qquad r_{n/n+2}\left(Q^2\right)=\frac{\mathcal{M}_n}{\mathcal{M}_{n+2}}, \tag{7}$$

where the experimental sides are more precise than the absolute moments $\mathcal{M}_n$. Also, in the ratios, partial cancellations of different perturbative as well as non-perturbative terms occur, which render the QCD approximation more precise than in the absolute moments.

The QCD sides of the sum rules are known in the literature since the original works of SVZ [1]. Their expressions at the subtraction scale $\nu^2=m_Q^2$ are given explicitly numerically in the appendix of [15] to 3-loops ($\mathcal{O}(\alpha_s^2)$) accuracy in terms of the running heavy quark mass[4] using the pQCD results of [32], while the $Q^2=0$ moments to 4-loops ($\mathcal{O}(\alpha_s^3)$) are given in [33] using the pQCD results in [34,35].[5] Among the different moments given in [15], we shall select three moments where both the $(\alpha_s)^n$ ($n=1,2$), the gluon condensate contributions and the effects of the higher resonances plus the QCD continuum are relatively small but not negligible. These conditions can be simultaneously satisfied by the moments[6]:

$$\begin{aligned}&\mathcal{M}_{2,3,4}\quad \text{for } Q^2=0,\\ &\mathcal{M}_{8,9,10}\quad \text{for } Q^2=4m_Q^2,\\ &\mathcal{M}_{13,14,15}\quad \text{for } Q^2=8m_Q^2.\end{aligned} \tag{8}$$

One may also work with more moments but these will not bring newer informations. Lower moments are more sensitive to the experimental errors and to the QCD continuum while higher moments are more sensitive to higher dimension condensates which are not under a good control.[7] Moreover, one can also note from the QCD expressions given by [15] that for n larger than in our previous selected choice, the signs of the pQCD corrections start to change compared to the original ones of the two-point correlator. A such change may introduce some systematical difficulties inherent to the approach for controlling the size of higher order terms.[8]

We shall work with the ratios of moments:

$$\begin{aligned}&r_{2/3}\quad\text{and}\quad r_{2/4}\quad \text{for } Q^2=0,\\ &r_{8/9}\quad\text{and}\quad r_{8/10}\quad \text{for } Q^2=4m_c^2,\\ &r_{13/14}\quad\text{and}\quad r_{13/15}\quad \text{for } Q^2=8m_c^2\end{aligned} \tag{9}$$

and use as inputs, in this first step:

$$\overline{m}_c(m_c)=1.26(3)\ \text{GeV}, \tag{10}$$

as given by different approaches using charmonium moments sum rules [1,3–5,15,31,33,41–43] and which we shall re-estimate later on. We shall also use:

$$\alpha_s(M_\tau)=0.3249(80)\quad\Longrightarrow\quad \alpha_s(m_c)|_{n_f=4}=0.408(14) \tag{11}$$

from τ-decay [14]; a value which agrees with the central value of the world average [31,44] when runned until M_Z.

The QCD expressions of the moments and their ratios are given in Table 2.

3. ρ_c from charmonium ratios of moments

We shall work with the ratios of moments in Eq. (9). We parametrize the spectral function by a sum of the six J/ψ-like

[4] We shall use these expressions in our analysis and we shall correct our final results on the quark masses by adding an estimate of the 4-loops ($\mathcal{O}(\alpha_s^3)$) contributions.

[5] Some Pade approximants are given in [36].

[6] However, one should note that the accuracy of the $Q^2=0$ moments is less than that of the $Q^2\neq 0$ moments, while one cannot use higher moments due to the bad convergence of the OPE in this case.

[7] The contributions of the dimension-8 condensates have been evaluated in [37] and can be sizeable if one assumes factorization which might not be applied here [38].

[8] In [39], some low energy gluon contributions to order α_s^3 to the correlator can invalidate the uses of the $Q^2=0$-moments for $n>4$ (see however [40]).