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# Model building by coset space dimensional reduction in eight-dimensions

Toshifumi Jittoh, Masafumi Koike, Takaaki Nomura *, Joe Sato, Yutsuki Toyama

*Department of Physics, Saitama University, Shimo-Okubo, Sakura-ku, Saitama 355-8570, Japan*

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ABSTRACT

We investigate gauge-Higgs unification models in eight-dimensional spacetime where extra-dimensional space has the structure of a four-dimensional compact coset space. The combinations of the coset space and the gauge group in the eight-dimensional spacetime of such models are listed. After the dimensional reduction of the coset space, we identified SO(10), SO(10) × U(1) and SO(10) × U(1) × U(1) as the possible gauge groups in the four-dimensional theory that can accomodate the Standard Model and thus is phenomenologically promising. Representations for fermions and scalars for these gauge groups are tabulated.



## 1. Introduction

The Standard Model has been eminently successful in describing the interactions of the elementary particles. A crucial role of this model is played by the Higgs scalar, which develops the vacuum expectation value to give the masses to the elementary particles and to trigger the breaking of the gauge symmetry from $SU(3)_C \times SU(2)_L \times U(1)_Y$ down to $SU(3)_C \times U(1)_{EM}$. On the other hand, the most fundamental nature of the Higgs scalar such as its mass is not predictable within the Standard Model. Thus, the search for the nature of this particle is essential both for the confirmation of the Standard Model and for the search for new physics.

The gauge-Higgs unification is an attractive approach to account for the origin of the Higgs scalars [1–3] (for recent approaches, see Refs. [4–20]). This approach counts the Higgs scalars as components of the gauge bosons in the spacetime with the dimension higher than four, and attributes their properties to the physical setups such as the gauge symmetry and the compactification scale of the extra-dimensional space. We consider this idea in the scheme of the coset space dimensional reduction, in which the extra-dimensional space is assumed to be a coset space of compact Lie groups, and the gauge transformation is identified as the translation within this space [1,21–26]. This identification determines both the gauge symmetry and the particle contents of the four-dimensional theory.

A phenomenologically promising gauge theory in a $D$-dimensional spacetime, where $D > 4$, should reproduce the Standard Model after the dimensional reduction. Theories in six- and ten-dimensional spacetime has attracted much attention so far in this regard. The chiral structure of the matter content as in the Standard Model is easy to introduce in these cases, more generally when $D = 4n + 2$ [27,28]. Fermions belonging to a vectorlike representation in $(4n + 2)$-dimensional gauge theory can end up in a chiral fermion after dimensional reduction by simultaneously applying the Weyl and the Majorana conditions, which are compatible in this dimensionality. This advantage increases the chance for the higher-dimensional model to be a promising candidate. No theories have been found quite promising, however, for the 6, 10, and 14 dimensional spacetimes [22,26,29–36].

We examine the theories in eight-dimensional spacetimes to search further for promising theories. The dimension of the extra-dimensional space $d = D - 4$ is four in this case, and the small $d$ makes the problem tractable. More importantly, the dimension $D = 8$ is below the critical dimension of the string theories, which may thus supply the ultraviolet completions to models in a spacetime of this dimensionality. On the other hand, we need to confine ourselves to the complex representations for the representations of the fermions, unlike the case of $D = 4n + 2$.

We search for the eight-dimensional gauge theory that leads to the Standard Model, the GUTs, or their likes. We exhaustively

* Corresponding author.
*E-mail addresses:* jittoh@phy.saitama-u.ac.jp (T. Jittoh), koike@phy.saitama-u.ac.jp (M. Koike), nomura@krishna.th.phy.saitama-u.ac.jp (T. Nomura), joe@phy.saitama-u.ac.jp (J. Sato), toyama@phy.saitama-u.ac.jp (Y. Toyama).

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search for the possible candidates of coset space $S/R$ and the gauge group $G$ of the eight-dimensional theory. The representation for the gauge bosons is then automatically determined. The representation of the fermions is searched up to 1000-dimensional ones, while even larger representations are avoided lest it should generate numerous unwanted fermions after the dimensional reduction. We also tabulate the representation of the scalars and fermions under the gauge group of the four-dimensional theory.

This Letter is organized as follows. In Section 2, we briefly recapitulate the scheme of the coset space dimensional reduction (CSDR) in eight dimensions. In Section 3, we search for the candidate of models in eight dimensions which lead to phenomenologically promising theories in four-dimensions after the dimensional reduction. Section 4 is devoted to summary and discussions.

## 2. CSDR scheme in eight dimensions

In this section, we briefly recapitulate the scheme of the coset space dimensional reduction in eight dimensions [22].

We begin with a gauge theory defined on an eight-dimensional spacetime $M^8$ with a simple gauge group $G$. Here $M^8$ is a direct product of a four-dimensional spacetime $M^4$ and a compact coset space $S/R$, where $S$ is a compact Lie group and $R$ is a Lie subgroup of $S$. The dimension of the coset space $S/R$ is thus $4 \equiv 8-4$, implying $\dim S - \dim R = 4$. This structure of extra-dimensional space requires the group $R$ be embedded into the group SO(4), which is a subgroup of the Lorentz group SO(1,7). Let us denote the coordinates of $M^8$ by $X^M = (x^\mu, y^\alpha)$, where $x^\mu$ and $y^\alpha$ are coordinates of $M^4$ and $S/R$, respectively. The spacetime index $M$ runs over $\mu \in \{0,1,2,3\}$ and $\alpha \in \{4,5,6,7\}$. In this theory, we introduce a gauge field $A_M(x,y) = (A_\mu(x,y), A_\alpha(x,y))$, which belongs to the adjoint representation of the gauge group $G$, and fermions $\psi(x,y)$, which lies in a representation $F$ of $G$.

The extra-dimensional space $S/R$ admits $S$ as an isometric transformation group. We impose on $A_M(X)$ and $\psi(X)$ the following symmetry under this transformation in order to carry out the dimensional reduction [21,37–41]. Consider a coordinate transformation which acts trivially on $x$ and gives rise to a $S$-transformation on $y$ as $(x,y) \to (x, sy)$, where $s \in S$. We require that the transformation of $A_M(X)$ and $\psi(X)$ under this coordinate transformation be compensated by a gauge transformation. This symmetry makes the eight-dimensional Lagrangian invariant under the $S$-transformation and therefore independent of the coordinate $y$ of $S/R$. The dimensional reduction is then carried out by integrating the eight-dimensional Lagrangian over the coordinate $y$ to obtain the four-dimensional one. The four-dimensional theory consists of the gauge fields $A_\mu$, fermions $\psi$, and in addition the scalar fields originated from $A_\alpha$. The gauge group reduces to a subgroup $H$ of the original gauge group $G$.

The gauge symmetry and particle contents of the four-dimensional theory are substantially constrained by the CSDR scheme. We provide below the prescriptions to identify the four-dimensional gauge group $H$ and its representations for the particle contents.

First, the gauge group of the four-dimensional theory $H$ is easily identified as

$$H = C_G(R), \tag{1}$$

where $C_G(R)$ denotes the centralizer of $R$ in $G$ [21]. Thus the four-dimensional gauge group $H$ is determined by the embedding of $R$ into $G$. These conditions imply

$$G \supset H \times R, \tag{2}$$

up to U(1) factors.

Second, the representations of $H$ for the scalar fields are specified by the following prescription. Let us decompose the adjoint representation of $S$ according to the embedding $S \supset R$ as

**Table 1**
A complete list of four-dimensional coset spaces $S/R$ with rank$S$ = rank$R$. We also list the decompositions of the vector representation **4** and the spinor representation $(\mathbf{2}_1,\mathbf{1}) + (\mathbf{1},\mathbf{2}_2)$ of $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ under the $R$s. The representations of $r_s$ in Eq. (3) and $\sigma_{1i}$ and $\sigma_{2i}$ in Eq. (6) are listed in the columns of "Branches of **4**" and "Branches of **2**", respectively.

| $S/R$ | | Branches of **4** | Branches of **2** |
|---|---|---|---|
| (i) | Sp(4)/[SU(2) × SU(2)] | $(\mathbf{2},\mathbf{2})$ | $(\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{2})$ |
| (ii) | SU(3)/[SU(2) × U(1)] | $\mathbf{2}(\pm1)$ | $\mathbf{2}(0)$ and $\mathbf{1}(\pm1)$ |
| (iii) | $(\mathrm{SU}(2)/\mathrm{U}(1))^2$ | $(\pm\mathbf{1},\pm\mathbf{1})$ | $(\pm\mathbf{1},0)$ and $(0,\pm\mathbf{1})$ |

$$\mathrm{adj}\, S = \mathrm{adj}\, R + \sum_s r_s. \tag{3}$$

We then decompose the adjoint representation of $G$ according to the embeddings $G \supset H \times R$;

$$\mathrm{adj}\, G = (\mathrm{adj}\, H, \mathbf{1}) + (\mathbf{1}, \mathrm{adj}\, R) + \sum_g (h_g, r_g), \tag{4}$$

where $r_g$s and $h_g$s denote representations of $R$ and $H$, respectively. The representation of the scalar fields are $h_g$s whose corresponding $r_g$s in the decomposition Eq. (4) are also contained in the set $\{r_s\}$ in Eq. (3).

Third, the representation of $H$ for the fermion fields is determined as follows [42]. The SO(1,7) Weyl spinor **8** is decomposed under its subgroup $(\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R})(\simeq \mathrm{SO}(1,3)) \times (\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2)(\simeq \mathrm{SO}(4))$ as

$$\mathbf{8} = (\mathbf{2}_L, \mathbf{1}, \mathbf{2}_1, \mathbf{1}) + (\mathbf{1}, \mathbf{2}_R, \mathbf{1}, \mathbf{2}_2), \tag{5}$$

where $(\mathbf{2}_L,\mathbf{1})$ and $(\mathbf{1},\mathbf{2}_R)$ representations of $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$ correspond to left- and right-handed spinors, respectively. The group $R$ is embedded into the Lorentz (SO(1,7)) subgroup SO(4) in such a way that the vector representation **4** of SO(4) is decomposed as $\mathbf{4} = \sum_s r_s$, where $r_s$s are the representations obtained in the decomposition Eq. (3). This embedding specifies a decomposition of the spinor representations $(\mathbf{2}_1,\mathbf{1})((\mathbf{1},\mathbf{2}_2))$ of $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \supset R$ as

$$(\mathbf{2}_1, \mathbf{1}) = \sum_i (\sigma_{1i}) \quad \left((\mathbf{1}, \mathbf{2}_2) = \sum_i (\sigma_{2i})\right). \tag{6}$$

We now decompose representation $F$ of the gauge group $G$ for the fermions in eight-dimensional spacetime. Decomposition of $F$ is

$$F = \sum_f (h_f, r_f), \tag{7}$$

under $G \supset H \times R$. The representations for the left-handed (right-handed) fermions are $h_f$s whose corresponding $r_f$s are found in $\sigma_{1i}(\sigma_{2i})$ obtained in Eq. (6).

A phenomenologically acceptable model needs chiral fermions in four dimensions as the SM does. The SO(1,7) spinor is not self-dual and its charge conjugate state is in a different representation from itself. Thus the Majorana condition cannot be used to obtain a chiral structure from a vectorlike representation of $G$. Therefore, we need to introduce complex representation for eight-dimensional fermions. Thus eight-dimensional model possesses a completely different feature from $(4n+2)$-dimensional models. We must work on complex representation for eight-dimensional fermions.

Finally coset space $S/R$ of our interest should satisfy rank$\,S$ = rank$\,R$ to generate chiral fermions in four dimensions [43]. We list all of four-dimensional coset spaces $S/R$ satisfying the condition and decompositions of SO(4) spinor and vector representation in Table 1.