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Extremal Kerr black hole/CFT correspondence in the five-dimensional Gödel universe

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ABSTRACT

We extend the method of Kerr/CFT correspondence recently proposed in arXiv:0809.4266 [hep-th] to the extremal (charged) Kerr black hole embedded in the five-dimensional Gödel universe. With the aid of the central charges in the Virasoro algebra and the Frolov–Thorne temperatures, together with the use of the Cardy formula, we have obtained the microscopic entropies that precisely agree with the ones macroscopically calculated by Bekenstein–Hawking area law.

1. Introduction

During the past decades, a lot of efforts have been devoted to studying the origin of Bekenstein-Hawking entropy for the black holes. Great progress has been made in their statistical interpretation thanks to Strominger and Vafa's remarkable work [1] on the investigation of the microscopic origin of the five-dimensional, supersymmetric (extremal) black hole entropy by using the holographic duality in the context of string theory. When the near-horizon limit has been taken, their work can be viewed as a typical example of the AdS/CFT correspondence [2–4], which shows that there exists a duality between the higher dimensional gravity and the CFT living on the boundary in less dimensions, providing a powerful tool to study the microscopic statistical mechanics of the black holes. By contrast, without using any supersymmetry, Strominger [5] has successfully evaluated the Bekenstein–Hawking entropy of the three-dimensional BTZ black hole by counting the number of the microstates in the two-dimensional CFT induced on the boundary of spatial infinity [6].

Quite recently, Guica, Hartman, Song and Strominger [7] put forward a new method called as Kerr/CFT correspondence to derive the microscopic entropy of the four-dimensional extremal Kerr black hole by identifying the quantum states in its near-horizon region with the two-dimensional chiral CFT on the spatially infinite boundary. The main ideas of their method go as follows: On the basis of the near-horizon geometry found in [8,9], one can construct diffeomorphisms that preserve a properly chosen boundary condition at the infinity. These diffeomorphisms generate one copy of the Virasoro algebra and contribute to the conserved charges. By computing the Dirac brackets of these charges, the central charge relative to the angular momentum of the extremal black hole can be obtained. Making use of the Frolov and Thorne temperature [10], the microscopic entropy in the dual CFT can be reproduced via the Cardy formula. Following this work, the microscopic entropies of the three-dimensional black hole and Kerr–AdS black holes in diverse dimensions were derived [11,12]. In Refs. [13,14], the Kerr/CFT correspondence was applied to the black holes with $U(1)$ gauge symmetry. Further extensions [15–18] has been made in (gauged) supergravity theory and string theory.

In this Letter, we shall apply the Kerr/CFT correspondence to the extremal (charged) Kerr black hole embedded in the five-dimensional Gödel universe [19,20], which is dubbed as a (charged) Kerr–Gödel black hole for shortness. These black hole metrics are exact solutions in the five-dimensional minimal supergravity. They possess some peculiar features such as the presence of closed time-like curves, and the absence of globally spatial-like Cauchy surface. Our Letter is organized as follows. In Section 2, we simply review the Kerr–Gödel black hole and obtain its near-horizon metric under the extremal condition. Based upon the near-horizon metric, we then calculate the central charge and mi-

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croscopic entropy of the extremal Kerr–Gödel black hole in the chiral dual CFT. In Section 3, we extend the analysis to the extremal charged Kerr–Gödel black hole. Finally, in Section 4, we end up with our conclusions.

2. Extremal Kerr–Gödel black hole and the dual CFT

In this section, we will generalize the method developed in [7, 14] to explore the duality between the extremal Kerr–Gödel black hole [19] and the chiral CFT by showing the equality of the microscopic CFT entropy and the Bekenstein–Hawking entropy. We first give a brief review of the Kerr–Gödel black hole solution [19] and then study its near-horizon geometry. Let's start with the metric

$$\begin{aligned} ds^2 = {} & -\left(1-\frac{2\mu}{\hat r^2}\right)d\hat t^2 + \frac{d\hat r^2}{\Delta_{\hat r}} \\ & -4\left(j\hat r^2 + \frac{\mu a}{\hat r^2}\right)\left(\cos^2\theta\, d\hat\phi + \sin^2\theta\, d\hat\psi\right)d\hat t \\ & -4\hat r^2\left(j^2\hat r^2 + 2j^2\mu - \frac{\mu a^2}{2\hat r^4}\right)\left(\cos^2\theta\, d\hat\phi + \sin^2\theta\, d\hat\psi\right)^2 \\ & +\hat r^2\left(d\theta^2 + \cos^2\theta\, d\hat\phi^2 + \sin^2\theta\, d\hat\psi^2\right), \end{aligned} \tag{1}$$

and the gauge potential which takes the form

$$A = \sqrt{3}\, j\hat r^2\left(\cos^2\theta\, d\hat\phi + \sin^2\theta\, d\hat\psi\right), \tag{2}$$

where

$$\Delta_{\hat r} = 1 - \frac{2\mu}{\hat r^2} + \frac{16 j^2\mu^2}{\hat r^2} + \frac{8 j\mu a}{\hat r^2} + \frac{2\mu a^2}{\hat r^4}. \tag{3}$$

In the above, the parameters μ and a are related to the mass and the angular momenta, respectively, while j defines the scale of the Gödel background and is responsible for the rotation of the universe. Without loss of generality, we assume μ, a and j are all positive. The metric (1) describes the rotating black hole with two equal angular velocities in the five-dimensional Gödel universe. The angular velocities and the electro-static potential on the horizon are given by

$$\begin{aligned} & \Omega^H_{\hat\phi} = \Omega^H_{\hat\psi} = 2\left(j\hat r_H^4 + \mu a\right)/\eta, \\ & \Phi_H = -2\sqrt{3}\, j\hat r_H^2\left(j\hat r_H^4 + \mu a\right)/\eta, \end{aligned} \tag{4}$$

where the event horizon $\hat r_H$ and constant η read

$$\begin{aligned} \hat r_H^2 = {} & \mu - 4\mu j a - 8j^2\mu^2 \\ & + \mu\sqrt{\left(1-8\mu j^2\right)\left(1-8\mu j^2 - 8ja - 2\mu^{-1}a^2\right)}, \\ \eta = {} & \hat r_H^4 + 2\mu a^2 - 4j^2\hat r_H^6 - 8\mu j^2\hat r_H^4. \end{aligned}$$

The temperature and the entropy are

$$S = 2\pi^2\hat r_H\sqrt{\eta}, \qquad T_H = \frac{\hat r_H^2 - \mu + 4j\mu a + 8j^2\mu^2}{\pi\hat r_H\sqrt{\eta}}. \tag{5}$$

In [21], the conserved quantities such as the mass, the angular momenta and the electrical charge have been computed as

$$\begin{aligned} & M = \frac{3}{4}\pi\mu - 8\pi j^2\mu^2 - \pi j\mu a, \\ & J_{\hat\phi} = J_{\hat\psi} = \frac{1}{2}\pi\mu a - \pi j\mu a^2 - 4\pi a j^2\mu^2, \\ & \mathcal{Q} = 2\sqrt{3}\pi j\mu a. \end{aligned} \tag{6}$$

Treating the Gödel parameter j as a fixed constant, we find that the variation of the mass, the angular momenta and the electrical charge satisfy the differential form of the first law

$$dM = T_H\, dS + \Omega^H_{\hat\phi}\, dJ_{\hat\phi} + \Omega^H_{\hat\psi}\, dJ_{\hat\psi} + \Phi_H\, d\mathcal{Q}. \tag{7}$$

However, if we take j as a thermodynamical variable, a conjugate generalized force should be introduced [20] to fulfill the first law of the black hole thermodynamics.

Now we turn our attention to the analysis of the near-horizon geometry of the extremal Kerr–Gödel black hole. The extremity condition is

$$j = \frac{(\mu - r_0^2)\sqrt{2}}{4\mu^{3/2}}, \qquad a = \frac{r_0^2}{\sqrt{2\mu}}, \tag{8}$$

where r_0 is the horizon radius of the extremal black hole, which makes the temperature T_H vanish. Under the extremal condition (8), to obtain the near-horizon geometry of the Kerr–Gödel black hole, we perform the coordinate transformation as follows

$$\begin{aligned} & \hat r = r_0 + r_0\lambda r, \qquad \hat t = \frac{r_0(\mu + r_0^2)\sqrt{2(2\mu - r_0^2)}}{8\mu^{3/2}\lambda} t, \\ & \hat\phi = \phi + \frac{\sqrt{2\mu - r_0^2}}{4r_0\lambda} t, \qquad \hat\psi = \psi + \frac{\sqrt{2\mu - r_0^2}}{4r_0\lambda} t, \end{aligned} \tag{9}$$

and then take the scaling parameter λ to zero, thus sending the metric (1) to the form

$$\begin{aligned} ds^2 = {} & \frac{1}{4} r_0^2\left(-r^2\, dt^2 + \frac{dr^2}{r^2} + 4\, d\theta^2\right) \\ & - \frac{r_0^4(3\mu^2 - r_0^4)}{2\mu^3}\cos^2\theta\sin^2\theta\,(d\phi - d\psi)^2 \\ & + \frac{r_0^2(2\mu - r_0^2)(\mu + r_0^2)^2}{2\mu^3} \\ & \times\left[\cos^2\theta\,(d\phi + \alpha r\, dt)^2 + \sin^2\theta\,(d\psi + \alpha r\, dt)^2\right], \end{aligned} \tag{10}$$

where

$$\alpha = \frac{r_0(3\mu - r_0^2)}{2(\mu + r_0^2)\sqrt{2\mu - r_0^2}}. \tag{11}$$

The near-horizon metric (10) depicts a 3-sphere bundle over the AdS_2 space. It only partially covers the near-horizon geometry of the extremal Kerr–Gödel black hole (1). One can perform global coordinate transformation to the coordinates (t, r) in order to make the metric (10) overlay the whole space in a single patch [9,15].

By virtue of the conformal structure of the near-horizon metric (10), it is possible for us to compute the central charges in the chiral CFT. Since there exist two rotations corresponding to ϕ and ψ, respectively, when the-near horizon metric (10) is assumed to have a certain suitable boundary, it can be shown as did in [7] that this metric can possess two commuting diffeomorphisms

$$\begin{aligned} & \zeta_n^{(1)} = -e^{-\mathrm{i}n\phi}\partial_\phi - \mathrm{i}nre^{-\mathrm{i}n\phi}\partial_r, \\ & \zeta_n^{(2)} = -e^{-\mathrm{i}n\psi}\partial_\psi - \mathrm{i}nre^{-\mathrm{i}n\psi}\partial_r \quad (n = 0, \pm 1, \pm 2, \ldots), \end{aligned} \tag{12}$$

which preserve the chosen boundary and generate two copies of commuting Virasoro algebra

$$\mathrm{i}\left[\zeta_m^{(i)}, \zeta_n^{(j)}\right] = (m - n)\delta_{ij}\zeta_{m+n}^{(i)} \quad (i, j = 1, 2). \tag{13}$$

Each diffeomorphism $\zeta_m^{(i)}$ is associated to a conserved charge defined by [22–25]

$$Q_{\zeta_n^{(i)}} = \frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta_n^{(i)}}[h, g], \tag{14}$$

where $\partial\Sigma$ is a spatial slice that extends to the infinity and the 3-form $k_{\zeta_n^{(i)}}[h, g]$ is written as