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Massive gravitons trapped inside a hypermonopole

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ABSTRACT

We propose a regular classical field theory realisation of the Dvali–Gabadadze–Porrati mechanism by considering our universe to be the four-dimensional core of a seven-dimensional 't Hooft–Polyakov hypermonopole. We show the existence of metastable gravitons trapped in the core. Their mass spectrum is discrete, positive definite, and computed for various values of the field coupling constants: the resulting Newton gravity law is seven-dimensional at small and large distances but can be made four-dimensional on intermediate length scales. There is no need of a cosmological constant in the bulk, the spacetime is asymptotically flat and of infinite volume in the extra-dimensions. Confinement is achieved through the local positive curvature of the extra-dimensions induced by the monopole-forming fields and for natural values of the coupling constants of order unity.

1. Introduction

Gravity occupies a central role in high energy physics and cosmology. On one hand, the unification of the fundamental interactions in the context of String Theory suggests that we may live in a more than four-dimensional world [1,2]. On the other hand, the recent acceleration of our universe has been confirmed by different experiments and it is now a widely accepted important result of modern observational cosmology [3]. The idea that such an unexplained acceleration may be the signature of extra-dimensions has been intensively explored in the recent years [4–6]. In the Dvali–Gabadadze–Porrati (DGP) model, the extra-dimensions (bulk) may actually be non-compact and of infinite volume [7,8]. Gravitons are reflected back onto our universe (brane) due to a different gravity coupling constant on the brane and in the bulk. The original DGP action in n_c+4 dimensions reads

$$S=\frac{M_{\mathrm{Pl}}^2}{2}\int\left|\sqrt{\bar{g}}\right|\bar{R}\,\mathrm{d}^4x+\frac{M_*^{2+n_c}}{2}\int\sqrt{|g|}R\,\mathrm{d}^{n_c+4}X, \tag{1}$$

where $\bar{g}$ and $\bar{R}$ are respectively the determinant and scalar curvature of the induced metric along our brane, while g and R are the corresponding quantities in the bulk. It has been shown that this model could actually explain the observed acceleration of the universe, although some works suggest that it may be spoiled by instabilities [9–12]. Although the form of Eq. (1) has been originally explained by quantum effects, "regularised models" have been proposed to justify it from a more classical and tractable point of view, free of instabilities. Such an approach has been explored in Refs. [13–15] and shown to confine gravitons by explicitly choosing some profile for $M_*(X)$ or $g(X)$. In a complete physical framework, both of these functions are however not free and it is not clear that the DGP mechanism could indeed appear in any classical system. This question is of crucial importance in order to assess the viability of both infinite volume extra-dimensions and instability-free DGP-like mechanism.

In this Letter, we answer this question in the context of canonical classical field theory. Our approach is motivated by condensed matter physics: topological defects are a direct consequence of the symmetry breaking mechanism and can model smooth branes [16–18]. Assuming the spacetime to be seven-dimensional, an $SO(3)$ spontaneous symmetry breaking in $n_c=3$ codimensions generically forms 't Hooft–Polyakov hypermonopoles [19,20]. In the following, we prove the existence of a DGP-like mechanism in the core (assumed to be our universe) of such a monopole.

Compared to lower-dimensional defects [21], the existence of positively curved (n_c-1)-dimensional regions in the bulk is crucial to allow metastable gravitons to be trapped inside the core. Six is indeed the minimal number of spatial dimensions for which there exists a foliation of the extra-dimensions by two-dimensional pos-

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itively curved surfaces. In order to allow for a varying Planck mass, we have for completeness included a dilaton ψ having a mass m_{d} in the Einstein frame. In the Jordan frame, the action associated with this system is

$$S = \frac{1}{2\kappa^2}\int \mathrm{e}^{\psi}\sqrt{-g}\left[R - g^{AB}\partial_A\psi\,\partial_B\psi - U(\psi)\right]\mathrm{d}^7x + \int\sqrt{-g}\left[-\frac{1}{2}g^{AB}\mathcal{D}_A\boldsymbol{\Phi}\cdot\mathcal{D}_B\boldsymbol{\Phi} - \frac{1}{4}\boldsymbol{H}_{AB}\cdot\boldsymbol{H}^{AB} - \frac{\lambda}{8}\left(\boldsymbol{\Phi}\cdot\boldsymbol{\Phi} - v^2\right)^2\right]\mathrm{d}^7x, \tag{2}$$

where the dilaton potential reads $U = m_{\mathrm{d}}^2\psi^2\exp(2\psi/5)$. The $SO(3)$ Higgs field $\boldsymbol{\Phi} = \{\phi^a\}$ is in the triplet representation ($a \in \{1, 2, 3\}$). Its vacuum expectation value v breaks $SO(3)$ into $U(1)$. The covariant derivatives $\mathcal{D}_A$ enforce gauge invariance and incorporate the gauge fields $\boldsymbol{C}_A = \{C_A^a\}$,

$$\mathcal{D}_A\boldsymbol{\Phi} = \partial_A\boldsymbol{\Phi} - q\boldsymbol{C}_A \wedge \boldsymbol{\Phi}, \tag{3}$$

q being the charge, while the field strength tensor $\boldsymbol{H}_{AB}$ is

$$\boldsymbol{H}_{AB} = \partial_A\boldsymbol{C}_B - \partial_B\boldsymbol{C}_A - q\boldsymbol{C}_A \wedge \boldsymbol{C}_B. \tag{4}$$

As for the dimensional analysis, we have $[\kappa^2] = M^{-5}$, $[q] = M^{-3/2}$, $[\lambda] = M^{-3}$ and $[C_A] = [\Phi] = [v] = M^{5/2}$.

2. Background geometry

Static self-gravitating monopole configurations associated with the action (2) can be obtained by imposing isotropy in the extra-dimensions, plus Poincaré invariance along the four internal brane coordinates x^μ. Our ansatz for the metric is

$$\mathrm{d}s^2 = \mathrm{e}^{\sigma(r)}\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu + \mathrm{d}r^2 + \omega(r)^2\,\mathrm{d}\Omega^2, \tag{5}$$

where r, θ, φ are spherical coordinates in three extra-dimensions and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$. For the monopole-forming fields, the internal space of $SO(3)$ is mapped to three extra-dimensions with a purely radial Higgs field

$$\boldsymbol{\Phi} = v f(r)\boldsymbol{u}_r, \tag{6}$$

and winding gauge fields

$$\boldsymbol{C}_\theta = \frac{1 - Q(r)}{q}\boldsymbol{u}_\varphi, \qquad \boldsymbol{C}_\varphi = -\frac{1 - Q(r)}{q}\sin\theta\,\boldsymbol{u}_\theta, \tag{7}$$

the other components vanishing. Here, $f(r)$ and $Q(r)$ are two dimensionless functions such that, far from the core, $f(r) \to 1$ and $Q(r) \to 0$ to recover a Dirac monopole. In the core, regularity imposes $f(0) = 0$ and $Q(0) = 1$. Concerning the metric coefficients, the energy associated with the defect being finite and localised, we look for asymptotically flat spacetime, $\sigma \to 0$, $\omega \to r$ and $\psi \to 0$. Regularity in the core also imposes $\sigma'(0) = \psi'(0) = 0$ and $\omega \sim r$.

The system of coupled non-linear differential equations obtained from the action (2) is of order ten and does not have any obvious analytical solution. Once the radial coordinate is expressed in unit of the Higgs Compton wavelength, the differential system is parametrised by three dimensionless parameters

$$\alpha \equiv \kappa^2 v^2, \qquad \epsilon \equiv \frac{q^2 v^2}{\lambda v^2} = \frac{m_{\mathrm{b}}^2}{m_{\mathrm{h}}^2}, \qquad \beta \equiv \frac{m_{\mathrm{d}}^2}{\lambda v^2} = \frac{m_{\mathrm{d}}^2}{m_{\mathrm{h}}^2}, \tag{8}$$

where m_{h} and m_{b} are respectively the mass of the Higgs and gauge bosons. Under the above-mentioned boundary conditions, the numerical integration of the equations of motion is a challenging problem that has been overcome by using recent advances in the field [22]. We have found monopole solutions for almost any values of the above parameters; only when the stress energy becomes super-Planckian the system develops some singularities preventing the spacetime to be asymptotically flat. As can be seen in Fig. 1, the Higgs and gauge field profiles are typical of topological defect configurations while the dilaton is gravitationally trapped inside the core. The profile of $\sigma(r)$ traces the gravitational redshift: clocks are ticking differently inside and outside the monopole. More interesting is the profile of $\omega(r)$. Up to a 4π factor, $\omega^2(r)$ gives the area of the two-sphere of radius r in the extra-dimensions. As can be seen in Fig. 1, there is a region at finite distance from the core where $\omega(r)$ does no longer grow as r but remains almost stationary: the extra-dimensions become cylindrically shaped. As we show in the next section, gravitons become resonant at these length scales and metastable from a four-dimensional point of view. Notice that the spacetime is non-compact and asymptotically Minkowski.

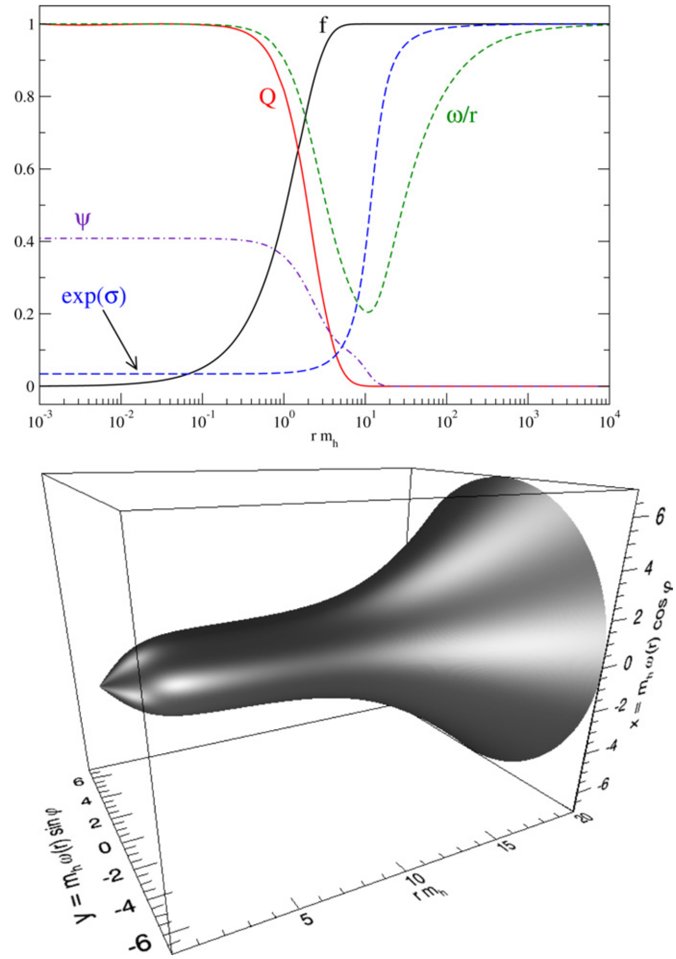

Fig. 1. Field and metric profiles forming the hypermonopole for $\alpha = 2.05$, $\epsilon = 0.50$ and $\beta = 1.00$ (top). The spacetime is flat asymptotically and in the core, but strongly curved in the intermediate region. The spatial sections for $\theta = \pi/2$ are represented in the bottom panel as a function of the radial coordinate.

3. Tensor fluctuations

We now consider the four-dimensional tensor perturbations around the previously computed background. The perturbed metric is given by Eq. (5) upon the replacement $\eta_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is a spacetime dependent transverse and traceless tensor. The linearised equations of motion for $h_{\mu\nu}$ are obtained by expanding Eq. (2) at second order and have already been derived