Physics Letters B 690 (2010) 250–254

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Effect of electric field of the electrosphere on photon emission from quark stars

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ARTICLE INFO

Article history:
Received 31 March 2010
Received in revised form 8 May 2010
Accepted 18 May 2010
Available online 21 May 2010
Editor: J.-P. Blaizot

Keywords:
Photon emission
Relativistic plasma
Quark stars

ABSTRACT

We investigate the photon emission from the electrosphere of a quark star. It is shown that at temperatures $T \sim 0.1$–1 MeV the dominating mechanism is the bremsstrahlung due to bending of electron trajectories in the mean Coulomb field of the electrosphere. The radiated energy flux from this mechanism exceeds considerably both the contribution from the bremsstrahlung due to electron–electron interaction and the tunnel e^+e^- pair creation.

1. It is possible that quark stars made of a stable strange quark matter (SQM) [1–3] (if it exists) may exist without a crust of normal matter [4]. The quark density for bare quark stars should drop abruptly at the scale ~ 1 fm. The SQM in normal phase and in the two-flavor superconducting (2SC) phase should also contain electrons (for normal phase the electron chemical potential, μ, is about 20 MeV [2,5]). Contrary to the quark density the electron density drops smoothly above the star surface at the scale $\sim 10^3$ fm [2,5]. For the star surface temperature $T \ll \mu$, say $T \lesssim 10^{10}$ K ~ 1 MeV, this "electron atmosphere" (usually called the electrosphere) may be viewed as a strongly degenerate relativistic electron gas [2,5]. The photon emission from the normal SQM is negligibly small as compared to the black body one at $T \ll \omega_p$ [6,7] (here $\omega_p \sim$ 20 MeV is the plasma frequency of the SQM [6]). However, for the electrosphere the plasma frequency is much smaller than that for the SQM. For this reason the photon emission from the electrosphere may potentially dominate the luminosity of a quark star. Contrary to neutron stars (or quark stars with a crust of normal matter) the photon emission from the electrosphere of bare quark stars may exceeds the Eddington limit, and may be used for distinguishing a bare quark star from a neutron star (or a quark star with a crust of normal matter). For this reason it is of great importance to have quantitative predictions for the photon emission from the electrosphere. This is also of interest in the context of the scenario of the gamma-ray repeaters due to reheating of a quark star by impact of a massive comet-like object [8].

The bremsstrahlung from the electrosphere due to the electron–electron interaction has been addressed in [9,10]. The authors of [9] used the soft photon approximation and factorized the $e+e \to e+e$ cross section in the spirit of Low's theorem. In [10] it was pointed out that this approximation is inadequate since it neglects the effect of the photon energy on the electron Pauli-blocking which should lead to a strong overestimate of the radiation rate. The authors of [10] have not given a consistent treatment of this problem either. To take into account the effect of the minimal photon energy they suggested some restrictions on the initial electron momenta introduced by hand. In this way they obtained the radiated energy flux from the $e^-e^- \to e^-e^-\gamma$ process which is much smaller than that in [9], and than the energy flux from annihilation of positrons produced in the tunnel e^+e^- creation in the electric field of the electrosphere [4,11]. In [12] there was an attempt to include the effect of the mean Coulomb field of the electrosphere on the photon emission. The authors obtained a considerable enhancement of the radiation rate. However, similarly to [9] the analysis [12] treats incorrectly the Pauli-blocking effect.

Thus the theoretical situation with the photon bremsstrahlung from the electrosphere is still controversial and uncertain. The main problem here is an accurate accounting for the photon energy in the Pauli-blocking. In the present Letter we address the bremsstrahlung from the electrosphere in a way similar to the Arnold–Moore–Yaffe (AMY) [13] approach to the collinear photon emission from a hot quark–gluon plasma within the thermal field theory. We use a reformulation of the AMY formalism given in [14] which is based on the light-cone path integral (LCPI) approach [15–17] (for reviews, see [18,19]) to the radiation processes. For an infinite homogeneous plasma (with zero mean field) the formalism [14] reproduces the AMY results [13]. The LCPI formulation [14] has the advantage that it also works for plasmas with nonzero mean field. It allows to evaluate the photon emission accounting

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0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.05.039

for bending of the electron trajectories in the mean Coulomb potential of the electrosphere. Contrary to very crude and qualitative methods of [9,10,12] the treatment of the Pauli-blocking effects in [13,14] has robust quantum field theoretical grounds. Of course, our approach is only valid in the regime of collinear photon emission when the dominating photon energies exceed several units of the photon quasiparticle mass. Numerical calculations show that even at $T \sim 0.1$ MeV the effect of the noncollinear configurations is relatively small.

We demonstrate that for the temperatures $T \sim 0.1$–1 MeV the radiated energy flux from the $e^- \to e^-\gamma$ transition in the mean electric field turns out to be much bigger than contributions from the $e^-e^- \to e^-e^-\gamma$ process and the tunnel e^+e^- creation. Our results show that the photon emission from the electrosphere may be of the same order as the black body radiation. For this reason the situation with distinguishing a bare quark star made of the SQM in normal (or 2SC) phase from a neutron star using the luminosity [4,20] may be more optimistic than in the scenario with the tunnel e^+e^- creation [4].

2. As in [4,9,10] we use for the electrosphere the model of a relativistic strongly degenerate electron gas in the Thomas–Fermi approximation. Then the electron chemical potential (related to the electrostatic potential, V, as $\mu = eV$) may be written as [2,5]

$$\mu(h) = \frac{\mu(0)}{(1+h/H)}, \tag{1}$$

where h is the distance from the quark surface, and $H = \sqrt{3\pi/2\alpha}/\mu(0)$, $\alpha = e^2/4\pi$ (we use units $c = \hbar = k_B = 1$).

We assume that the electrosphere is optically thin. Then the luminosity may be expressed in terms of the energy radiated spontaneously per unit time and volume, Q_γ, usually called the emissitivity. In the formalism [14] the emissitivity per unit photon energy ω at a given h can be written as

$$\frac{dQ_\gamma(h,\omega)}{d\omega} = \frac{\omega(k)}{4\pi^3}\frac{dk}{d\omega} \times \int \frac{d\mathbf{p}}{p} n_F(E)\big[1-n_F(E')\big]\theta(p-k)\frac{dP(\mathbf{p},x)}{dx\,dL}, \tag{2}$$

where k is the photon momentum, E and E' are the electron energies before and after photon emission, $n_F(E) = (\exp((E-\mu)/T)+1)^{-1}$ is the local electron Fermi distribution (we omit the argument h in the functions on the right-hand side of (2)), $x = k/p$ is the photon longitudinal (along the initial electron momentum $\mathbf{p}$) fractional momentum. The function $dP/dx\,dL$ in (2) is the probability of the photon emission per unit x and length from an electron in the potential generated by other electrons which includes both the smooth collective Coulomb field and the usual fluctuating part. Note that (2) assumes that the photon emission is a local process, *i.e.* the photon formation length l_f is small compared to the thickness of the electrosphere.

In the LCPI formalism [15,18] the photon spectrum $dP/dx\,dL$ can be written as

$$\frac{dP}{dx\,dL} = 2\,\mathrm{Re}\int_0^\infty d\xi\,\hat{g}(x)\big[\mathcal{K}(\boldsymbol{\rho}',\xi|\boldsymbol{\rho},0) - \mathcal{K}_v(\boldsymbol{\rho}',\xi|\boldsymbol{\rho},0)\big]\Big|_{\boldsymbol{\rho}'=\boldsymbol{\rho}=0}. \tag{3}$$

Here $\hat{g}$ is the spin vertex operator (it can be found in [18]), $\mathcal{K}$ is the Green's function for the two-dimensional Hamiltonian

$$\hat{H} = -\frac{1}{2M(x)}\left(\frac{\partial}{\partial\boldsymbol{\rho}}\right)^2 + v(\boldsymbol{\rho}) + \frac{1}{L_0}, \tag{4}$$

where $M(x) = px(1-x)$, $L_0 = 2M(x)/\epsilon^2$, $\epsilon^2 = m_e^2x^2 + (1-x)m_\gamma^2$ (m_γ is the photon quasiparticle mass), the form of the potential v will be given below. In (3), (4) $\boldsymbol{\rho}$ is the coordinate transverse to the electron momentum $\mathbf{p}$, the longitudinal (along $\mathbf{p}$) coordinate ξ plays the role of time. The $\mathcal{K}_v$ in (3) is the free Green's function for $v = 0$. Note that at low density and vanishing mean field the quantity L_0 coincides with the real photon formation length l_f [15].

The potential in the Hamiltonian (4) can be written as $v = v_m + v_f$. The terms v_m and v_f correspond to the mean and fluctuating components of the vector potential of the electron gas. Note that when l_f is small compared to the scale of variation of μ (along the electron momentum) one can neglect the ξ-dependence of the potential v in evaluating $dP/dx\,dL$. The mean field component is purely real $v_m = -x\mathbf{f}\cdot\boldsymbol{\rho}$ with $\mathbf{f} = e\partial V/\partial\boldsymbol{\rho}$ [18,21]. It is related to the transverse force from the mean field. Note that, similarly to the classical radiation [22], the effect of the longitudinal force along the electron momentum $\mathbf{p}$ is suppressed by a factor $\sim (m_e/E)^2$, and can be safely neglected. The term v_f can be evaluated similarly to the case of the quark–gluon plasma discussed in [14]. This part is purely imaginary $v_f(\boldsymbol{\rho}) = -iP(x\boldsymbol{\rho})$, where

$$P(\boldsymbol{\rho}) = e^2\int_{-\infty}^{\infty} d\xi\,\big[G(\xi,0_\perp,\xi) - G(\xi,\boldsymbol{\rho},\xi)\big], \tag{5}$$

$G(x-y) = u_\mu u_\nu D^{\mu\nu}$, $D^{\mu\nu} = \langle A^\mu(x)A^\nu(y)\rangle$ is the correlation function of the electromagnetic potential (the mean field is assumed to be subtracted) in the electron plasma, $u_\mu = (1,0,0,-1)$ is the light-cone 4-vector (along the electron momentum). The correlator $D^{\mu\nu}$ may be expressed in terms of the longitudinal and transverse photon self-energies, $\Pi_{L,T}$ [13]. In numerical calculations we use for the $\Pi_{L,T}$ the well-known hard dense loop expressions [23,24].

Treating v_f as a perturbation one can write

$$\mathcal{K}(\xi_2,\boldsymbol{\rho}_2|\xi_1,\boldsymbol{\rho}_1) = \mathcal{K}_m(\xi_2,\boldsymbol{\rho}_2|\xi_1,\boldsymbol{\rho}_1) - i\int d\xi\,d\boldsymbol{\rho}\,\mathcal{K}_m(\xi_2,\boldsymbol{\rho}_2|\xi,\boldsymbol{\rho})v_f(\boldsymbol{\rho})\mathcal{K}_m(\xi,\boldsymbol{\rho}|\xi_1,\boldsymbol{\rho}_1) + \cdots, \tag{6}$$

where $\mathcal{K}_m$ is the Green's function for $v_f = 0$. Then (3) can be written as

$$\frac{dP}{dx\,dL} = \frac{dP_m}{dx\,dL} + \frac{dP_f}{dx\,dL}. \tag{7}$$

Here the first term on the right-hand side comes from the $\mathcal{K}_m - \mathcal{K}_v$ in (3) after representing $\mathcal{K}$ in the form (6). It corresponds to the photon emission in a smooth mean field. The second term comes from the series in v_f in (6). This term can be viewed as the radiation rate due to electron multiple scattering in the fluctuating field in the presence of a smooth external field. The analytical expression for the Green's function $\mathcal{K}_m$ is known (see, for example [25]). The corresponding spectrum is similar to the well-known synchrotron spectrum, and can be written in terms of the Airy function $\mathrm{Ai}(z) = \frac{1}{\pi}\sqrt{\frac{z}{3}}K_{1/3}(2z^{3/2}/3)$ (here $K_{1/3}$ is the Bessel function) [21,26]. In the case of interest, for a nonzero photon quasiparticle mass it reads [21]

$$\frac{dP_m}{dx\,dL} = \frac{a}{\kappa}\mathrm{Ai}'(\kappa) + b\int_\kappa^\infty dy\,\mathrm{Ai}(y), \tag{8}$$

where $a = -2\epsilon^2 g_1/M$, $b = Mg_2 - \epsilon^2 g_1/M$, $\kappa = \epsilon^2/(M^2x^2\mathbf{f}^2)^{1/3}$, $g_1 = \alpha(1-x+x^2/2)/x$ and $g_2 = \alpha m_e^2x^3/2M^2$. Note that the effective photon formation length for the mean field mechanism is given by $\bar{L}_m \sim \min(L_0, L_m)$, where $L_m = (24M/x^2\mathbf{f}^2)^{1/3}$ [21].