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Lorentz violation and black-hole thermodynamics: Compton scattering process

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ABSTRACT

A Lorentz-noninvariant modification of quantum electrodynamics (QED) is considered, which has photons described by the nonbirefringent sector of modified Maxwell theory and electrons described by the standard Dirac theory. These photons and electrons are taken to propagate and interact in a Schwarzschild spacetime background. For appropriate Lorentz-violating parameters, the photons have an effective horizon lying outside the Schwarzschild horizon. A particular type of Compton scattering event, taking place between these two horizons (in the photonic ergoregion) and ultimately decreasing the mass of the black hole, is found to have a nonzero probability. These events perhaps allow for a violation of the generalized second law of thermodynamics in the Lorentz-noninvariant theory considered.

1. Introduction

Lorentz-violating theories coupled to gravity can have interesting black-hole solutions. Particles that obey Lorentz-violating dispersion relations may perceive an effective horizon different from the event horizon for standard Lorentz-invariant matter [1–3]. It has been argued [1,2] that such multiple-horizon structures allow for the construction of a perpetuum mobile of the second kind (involving heat transfer from a cold body to a hot body, without other change).

This Letter considers modified Maxwell theory [4] as a concrete realization of a Lorentz-violating theory. With an appropriate choice for the Lorentz-violating parameters, the nonstandard photons have an effective horizon lying outside the Schwarzschild event horizon for standard matter. Of interest, now, are Compton scattering events $\gamma e^- \to \gamma e^-$, which take place between these two horizons, that is, in the accessible part of the photonic ergosphere region. After the collision, the photon may carry negative Killing energy as it propagates inside the photonic ergosphere, so that the final electron carries away more Killing energy than the sum of the Killing energies of the ingoing particles. As shown in Section IV.B of Ref. [2], such a scattering event ultimately *reduces* the black-hole mass. In the following, it will be demonstrated that this particular Compton scattering event is kinematically allowed and has a nonvanishing probability to occur.

The purpose of this Letter is to give a concrete example of a Compton scattering event that can be used to reduce the black-hole mass. This requires a detailed discussion of the theory in Section 2, which can, however, be skipped in a first reading. The main result is presented in Section 3 and discussed in Section 4, both of which sections are reasonably self-contained.

2. Setup

2.1. Units and conventions

Natural units are used with $c = G_N = \hbar = 1$. Spacetime indices are denoted by Greek letters and correspond to t, r, θ, ϕ for standard spherical Schwarzschild coordinates or to τ, R, θ, ϕ for Lemaître coordinates. Local Lorentz indices are denoted by Latin letters and run from 0 to 3. The flat-spacetime Minkowski metric is η_{ab} and the curved-spacetime Einstein metric $g_{\mu\nu}$, both with signature $(+,-,-,-)$. The determinant of the metric is denoted by $g \equiv \det g_{\mu\nu}$. The vierbeins are introduced in the standard way by writing $g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}$ and obey the relations $e^\mu{}_a e_\mu{}^b = \delta_a^b$ and $e^\mu{}_a e_\nu{}^a = \delta_\nu^\mu$.

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2.2. Modified QED in curved spacetime

Modified Maxwell theory is an Abelian $U(1)$ gauge theory with a Lagrange density that consists of the standard Maxwell term and an additional Lorentz-violating bilinear term [4–7]. The vierbein formalism is particularly well-suited for describing Lorentz-violating theories in curved spacetime, since it allows to distinguish between local Lorentz and general coordinate transformations [8] and to set the torsion identically to zero.

A minimal coupling procedure then yields the following Lagrange density for the photonic part of the action:

$$\mathcal{L}_{\mathrm{modM}} = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{4}\kappa^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}, \tag{2.1a}$$

$$\kappa^{\mu\nu\rho\sigma} \equiv \kappa^{abcd} e^{\mu}{}_{a} e^{\nu}{}_{b} e^{\rho}{}_{c} e^{\sigma}{}_{d}, \tag{2.1b}$$

in terms of the standard Maxwell field strength tensor $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$. The "tensor" κ^{abcd} has the same symmetries as the Riemann curvature tensor, as well as a double-trace condition. The numbers $\kappa^{abcd}(x)$ are considered to be fixed parameters, with no field equations of their own.

In the following, we explicitly choose this background tensor field to be of the form [5]

$$\kappa^{abcd}(x) = \frac{1}{2}\big(\eta^{ac}\tilde{\kappa}^{bd}(x) - \eta^{ad}\tilde{\kappa}^{bc}(x) + \eta^{bd}\tilde{\kappa}^{ac}(x) - \eta^{bc}\tilde{\kappa}^{ad}(x)\big), \tag{2.2}$$

in terms of a symmetric and traceless background field $\tilde{\kappa}^{ab}(x)$. Physically, (2.2) implies the restriction to the nonbirefringent sector of modified Maxwell theory. Moreover, we employ the following decomposition of $\tilde{\kappa}^{ab}(x)$:

$$\tilde{\kappa}^{ab}(x) = \kappa\big(\xi^a(x)\xi^b(x) - \eta^{ab}/4\big), \tag{2.3}$$

relative to a normalized parameter four-vector ξ^a with $\xi_a\xi^a = 1$. For our purpose, we will choose the parameter κ in (2.3) to be spacetime independent.

The breaking of Lorentz invariance in the electromagnetic theory (2.1) is indicated by the fact that the flat-spacetime theory allows for maximal photon velocities different from $c = 1$ (operationally defined by the maximum attainable velocity of standard Lorentz-invariant particles to be discussed shortly). See, e.g., Refs. [4–7] for further details of the simplest version of modified Maxwell theory with constant κ^{abcd} over Minkowski spacetime and physical bounds on its 19 parameters.

The charged particles (electrons) are described by the standard Dirac Lagrangian over curved spacetime [9] and gravity itself by the standard Einstein–Hilbert Lagrangian [10]. All in all, this particular modification of quantum electrodynamics (QED) has action

$$S = \int_{\mathbb{R}^4} d^4x\, \sqrt{-g}\,(\mathcal{L}_{\mathrm{EH}} + \mathcal{L}_{\mathrm{D}} + \mathcal{L}_{\mathrm{modM}}), \tag{2.4a}$$

$$\mathcal{L}_{\mathrm{EH}} = R/(16\pi), \tag{2.4b}$$

$$\mathcal{L}_{\mathrm{D}} = \bar{\psi}\left(\frac{1}{2}\gamma^a e^{\mu}{}_{a}\, \mathrm{i}\overleftrightarrow{\nabla}_{\mu} - m\right)\psi, \tag{2.4c}$$

with Ricci curvature scalar R from the metric $g_{\mu\nu}$, the usual Dirac matrices γ^a, and the gauge- and Lorentz-covariant derivative of a spinor [9],

$$\nabla_\mu\psi \equiv \partial_\mu\psi + \Gamma_\mu\psi - eA_\mu\psi, \tag{2.5a}$$

with spin connection

$$\Gamma_\mu = \frac{1}{2}\Sigma^{ab} e_a{}^{\nu}\partial_\mu(e_{b\nu}), \qquad \Sigma_{ab} \equiv \frac{1}{4}(\gamma_a\gamma_b - \gamma_b\gamma_a). \tag{2.5b}$$

2.3. Effective background for the photons

As demonstrated in Section 3 of Ref. [3], photons described by the Lagrange density (2.1) with the Lorentz-violating parameters (2.2)–(2.3) propagate on null-geodesics of an effective metric. This effective metric is given by:

$$\tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x) - \frac{\kappa}{1+\kappa/2}\xi_\mu(x)\xi_\nu(x), \tag{2.6}$$

with an inverse following from $\tilde{g}^{\mu\nu}\tilde{g}_{\nu\rho} = \delta^{\mu}{}_{\rho}$. All lowering or raising of indices is, however, understood to be performed by contraction with the original background metric $g_{\mu\nu}$ or its inverse $g^{\mu\nu}$, unless stated otherwise.

In order to avoid obvious difficulties with causality, we restrict our considerations to a subset of theories without space-like photon trajectories (with respect to the original metric). This is ensured by the choice $0 \leqslant \kappa < 2$.

2.4. Schwarzschild spacetime metric

In the following, we consider a standard Schwarzschild geometry as given by the following line element:

$$ds^2 = (1 - 2M/r)\,dt^2 - (1 - 2M/r)^{-1}\,dr^2 - r^2\,d\Omega^2, \tag{2.7a}$$

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta\,d\phi^2. \tag{2.7b}$$

It will be convenient to work with Lemaître coordinates,

$$ds^2 = d\tau^2 - \left(\frac{3(R-\tau)}{4M}\right)^{-2/3} dR^2 - \big(3/2(R-\tau)\big)^{4/3}(2M)^{2/3}\,d\Omega^2, \tag{2.8}$$

as Lemaître coordinates describe the standard Schwarzschild solution in coordinates which are nonsingular at the horizon (corresponding to the reference frame of a free-falling observer).

The transformation to standard Schwarzschild coordinates reads

$$d\tau = dt + \frac{\sqrt{2M/r}}{1 - 2M/r}\,dr, \tag{2.9a}$$

$$dR = dt + \frac{1}{(1 - 2M/r)\sqrt{2M/r}}\,dr, \tag{2.9b}$$

and the horizon is described by $(R - \tau) = (4/3)M$. A suitable choice of the vierbein $e_{\mu}{}^{a}$ is given by

$$e_\tau{}^0 = 1, \quad e_R{}^1 = \sqrt{|g_{RR}|}, \quad e_\theta{}^2 = \sqrt{|g_{\theta\theta}|}, \quad e_\phi{}^3 = \sqrt{|g_{\phi\phi}|}, \tag{2.10}$$

with all other components vanishing.

2.5. Effective Schwarzschild metric for the photons

For the vector field $\xi^\mu(x) = e^{\mu}{}_{a}(x)\xi^a(x)$ entering the nonstandard part of the photonic action (2.1)–(2.3) and the effective Lorentz-violating parameter, we take

$$\xi^\mu(x) = (1, 0, 0, 0), \tag{2.11a}$$

$$\epsilon \equiv \frac{\kappa}{1 - \kappa/2}, \tag{2.11b}$$

where the first expression (in Lemaître coordinates) makes clear that the photonic Lorentz violation is isotropic and the last expression introduces a convenient Lorentz-violating parameter for the theory considered. The particular parameter choices (2.11) correspond to Case 1 in Ref. [3]. Asymptotically ($R \to \infty$ for fixed τ),