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Nonperturbative deformation of D-brane states by the world sheet noncommutativity

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ABSTRACT

By introducing the noncommutativity in the world sheet, we discuss a modification of the D-brane states in the closed string theory. In particular we show how the world sheet noncommutativity induces a nonperturbative effect to the D-brane states.

1. Introduction

The noncommutative spacetime provided by the string theory [1] has been discussed as a possible basis of the theory of quantum gravity [2]. In recent years, it has been shown that the noncommutative spaces can be constructed by the help of the concept of Hopf algebra [3] from certain commutative spaces which have Lie algebraic symmetries. We can consider the string theory, the D-brane dynamics and the M-theory on the noncommutative space, in which the noncommutativity plays an important role in the description of their nonperturbative aspects.

From this point of view, it will be interesting to investigate the effect of the noncommutativity of the string world sheet. In fact the two-dimensional noncommutative conformal field theories have been discussed in various aspects, such as the perturbative approach [4], the harmonic oscillator formalism [5] and the derivation of the star product by virtue of the Hopf algebra [6]. In spite of many efforts, however, we are still far from obtaining some interesting results.

One of the main difficulties of these approaches owes to the fact that the noncommutativity of the world sheet introduces a dimensional constant which breaks explicitly the conformal symmetry of the theory. To recover the theory from this problem, one must commit oneself to many complicated formulae expanded into perturbative series.

On the other hand the world sheet noncommutativity possesses a notable feature, i.e., it does not affect the product of the left-movers, which are holomorphic, nor the product of the right-movers, which are anti-holomorphic, of the strings. Therefore the influence of the world sheet noncommutativity appears only in the product between the left-movers and the right-movers in the closed string theory. This means, for example, that the world sheet noncommutativity does not change the theory of only left-movers at all. Hence the world sheet noncommutativity is nothing to do with the spacetime quantization. In other words we must quantize the spacetime independent from the world sheet noncommutativity.

Considering the above background into account, we would like to study, in this note, some nonperturbative effects of the world sheet noncommutativity to the D-brane states. To this end we assume that all physical quantities can be represented by the string coordinates $X^\mu(\sigma,\tau)$ and that the effect of the world sheet noncommutativity appears only through their products. Moreover we assume that, although the left-moving string coordinate $X^\mu(\sigma+i\tau)$ and the right-moving string coordinate $\tilde{X}^\mu(\sigma-i\tau)$ do not commute any more, their components α_n^μ and $\tilde{\alpha}_n^\mu$ commute, so that the world sheet noncommutativity does not change the conventional quantization rule of the spacetime. We will see that these assumptions enable us to put forward our investigation nonperturbatively. At the same time we must emphasize that we leave aside the problem of the breaking of the conformal symmetry.

In order to see the effect of the world sheet noncommutativity, the D-brane is a convenient object to study because the D-brane boundary state consists of a product of the left and right moving string coordinates. The star product associated with the world sheet noncommutativity is introduced locally. Nevertheless we will see that it produces a nonlocal deformation of the D-brane boundary state as a nonperturbative effect. In the D-brane correlators, for example, the noncommutativity parameter λ appears through the elliptic functions as their modular parameter. All such results suggest that the world sheet noncommutativity changes the topology of the world sheet itself.

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Since the D-brane is a much-discussed object and many results are accumulated, we should be able to compare our results with them. Although we do not even consider the fermionic partners and the ghosts in this note, our results are enough to see the effect of the world sheet noncommutativity.

The organization of this note is as follows. In Section 2, we define the star product and consider the noncommutativity in the world sheet. In Section 3, we evaluate correlators of the D-brane states as deformed by the world sheet noncommutativity. Finally, we summarize the results in Section 4.

2. World sheet noncommutativity

In two-dimensional space, the $*$-product can be represented by the differential operators of the coordinate as follows;

$$f(\sigma,\tau)*g(\sigma,\tau)=\lim_{(\sigma',\tau')\to(\sigma,\tau)} e^{-\lambda(\partial_\sigma\partial_{\tau'}-\partial_\tau\partial_{\sigma'})} f(\sigma,\tau)g(\sigma',\tau'), \tag{1}$$

with only one noncommutativity parameter λ. The commutator of the coordinates with respect to this product gives a constant value which means noncommutative space:

$$[\tau,\sigma]_*=2\lambda.$$

In order to investigate the physical aspects, however, we have to discuss the theory of fields at different points and so we give a versatility to the $*$-product which acts on multiple different points. In the case of only two points, we extend it by the copy of (1) without taking the limitation $(\sigma',\tau')\to(\sigma,\tau)$:

$$f(\sigma,\tau)*g(\sigma',\tau')=e^{-\lambda(\partial_\sigma\partial_{\tau'}-\partial_\tau\partial_{\sigma'})} f(\sigma,\tau)g(\sigma',\tau'). \tag{2}$$

Since this $*$-product is compatible with the trigonometric functions

$$e^{im\sigma+n\tau}*e^{im'\sigma'+n'\tau'}=e^{-i\lambda(mn'-nm')}e^{im\sigma+n\tau}e^{im'\sigma'+n'\tau'} \quad \forall m,n,m',n'\in\mathbb{Z},$$

it is possible to define any product of fields at different points through their Fourier expansion. In fact we can convince ourselves that the associativity

$$\begin{aligned}
&e^{im_1\sigma_1+n_1\tau_1}*\left(e^{im_2\sigma_2+n_2\tau_2}*e^{im_3\sigma_3+n_3\tau_3}\right)\\
&\quad=e^{-i\lambda(m_2n_3-n_2m_3)}e^{im_1\sigma_1+n_1\tau_1}*\left(e^{im_2\sigma_2+n_2\tau_2}e^{im_3\sigma_3+n_3\tau_3}\right)\\
&\quad=e^{-i\lambda(m_1n_2-n_1m_2)}e^{-i\lambda(m_2n_3-n_2m_3)}e^{-i\lambda(m_1n_3-n_1m_3)}e^{im_1\sigma_1+n_1\tau_1}e^{im_2\sigma_2+n_2\tau_2}e^{im_3\sigma_3+n_3\tau_3}\\
&\quad=\left(e^{im_1\sigma_1+n_1\tau_1}*e^{im_2\sigma_2+n_2\tau_2}\right)*e^{im_3\sigma_3+n_3\tau_3}
\end{aligned}$$

is preserved, which is the sufficient property for our purpose.

Now we focus our attention to the string theory. It is described by the string coordinate, which is either holomorphic or anti-holomorphic function of the world sheet coordinate (σ,τ). Due to this fact an evaluation of the contribution of the world sheet noncommutativity is not difficult.

Let $f_a(\sigma+i\tau)$ and $g_a(\sigma-i\tau)$ be a holomorphic and an anti-holomorphic functions which can be expanded into the series of the complex variable $z=e^{i(\sigma+i\tau)}$ and its conjugate, respectively, as follows

$$f_a(\sigma+i\tau)=\sum_{n\in\mathbb{Z}} f_{a,n}e^{in(\sigma+i\tau)}, \qquad g_a(\sigma-i\tau)=\sum_{n\in\mathbb{Z}} g_{a,n}e^{in(\sigma-i\tau)}.$$

Then the $*$-product (2) gives the following formulae,

$$f_a(\sigma+i\tau)*f_b(\sigma'+i\tau')=f_a(\sigma+i\tau)f_b(\sigma'+i\tau'), \tag{3}$$

$$g_a(\sigma-i\tau)*g_b(\sigma'-i\tau')=g_a(\sigma-i\tau)g_b(\sigma'-i\tau'), \tag{4}$$

$$f_a(\sigma+i\tau)*g_b(\sigma'-i\tau')=\int_0^{2\pi}\int_0^{2\pi}\frac{d\sigma_1\,d\sigma_2}{4\pi\lambda}e^{i(\sigma_1-\sigma)(\sigma_2-\sigma')/2\lambda}f_a(\sigma_1+i\tau)g_b(\sigma_2-i\tau'). \tag{5}$$

In the same point limit this formalism has been already mentioned in [7]. We learn, from these results, that any product of two holomorphic functions, as well as two anti-holomorphic functions, is not affected by the world sheet noncommutativity. In other words the world sheet noncommutativity does not violate the analytic properties of each field. We have to attend our concern to the product only between a holomorphic and an anti-holomorphic components.

On the basis of definitions above, we want to study the free closed string theory on the noncommutative world sheet. The spacetime coordinates of a closed string are defined by

$$X^\mu(\sigma+i\tau)=X_+^\mu(\sigma+i\tau)+X_-^\mu(\sigma+i\tau), \qquad \tilde{X}^\mu(\sigma-i\tau)=\tilde{X}_+^\mu(\sigma-i\tau)+\tilde{X}_-^\mu(\sigma-i\tau),$$

$$\begin{cases} X_+^\mu(\sigma+i\tau):=\frac{1}{2}x_0^\mu-i\sum_{n=1}^\infty\frac{1}{n}\alpha_{-n}^\mu e^{-in(\sigma+i\tau)},\\ X_-^\mu(\sigma+i\tau):=-2p_0^\mu(\sigma+i\tau)+i\sum_{n=1}^\infty\frac{1}{n}\alpha_n^\mu e^{in(\sigma+i\tau)},\end{cases}$$

$$\begin{cases} \tilde{X}_+^\mu(\sigma-i\tau):=\frac{1}{2}x_0^\mu-i\sum_{n=1}^\infty\frac{1}{n}\tilde{\alpha}_{-n}^\mu e^{in(\sigma-i\tau)},\\ \tilde{X}_-^\mu(\sigma-i\tau):=2p_0^\mu(\sigma-i\tau)+i\sum_{n=1}^\infty\frac{1}{n}\tilde{\alpha}_n^\mu e^{-in(\sigma-i\tau)},\end{cases}$$