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The neutrino response of low-density neutron matter from the virial expansion

C.J. Horowitz [a], A. Schwenk [b,*]

[a] Nuclear Theory Center and Department of Physics, Indiana University, Bloomington, IN 47408, USA
[b] Department of Physics, University of Washington, Seattle, WA 98195-1560, USA

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Abstract

We generalize our virial approach to study spin-polarized neutron matter and the consistent neutrino response at low densities. In the long-wavelength limit, the virial expansion makes model-independent predictions for the density and spin response, based only on nucleon–nucleon scattering data. Our results for the neutrino response provide constraints for random-phase approximation or other model calculations, and we compare the virial vector and axial response to response functions used in supernova simulations. The virial expansion is suitable to describe matter near the supernova neutrinosphere, and this work extends the virial equation of state to predict neutrino interactions in neutron matter.

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1. Introduction

Neutrinos radiate 99% of the energy in core-collapse supernovae. The scattering of neutrinos and the physics of the explosion are most sensitive to the properties of low-density nucleonic matter [1,2], which is a complex problem due to strong coupling with large scattering lengths, clustering in nuclear matter and the non-central nature of nuclear interactions. For low densities and high temperatures, the virial expansion provides a tractable approach to strong interactions, and in previous works we have presented the virial equation of state of low-density nucleonic matter [3,4]. The predicted large symmetry energy at low densities has been confirmed in near Fermi energy heavy-ion collisions [5].

The virial approach can be used to describe matter in thermal equilibrium around the neutrinosphere in supernovae. The temperature of the neutrinosphere is roughly $T \sim 4$ MeV from about 20 neutrinos detected in SN1987a [6,7], and the density follows from known cross sections of neutrinos with these energies $n \sim 10^{11}$–10^{12} g/cm^3. For neutron matter, the virial expansion in terms of the fugacity $z = e^{\mu/T}$ is valid for

$$n = \frac{2}{\lambda^3} z + \mathcal{O}(z^2) \lesssim 4 \times 10^{11}\ (T/\mathrm{MeV})^{3/2}\ \mathrm{g/cm^3}, \qquad (1)$$

where we require $z < 1/2$ and λ denotes the thermal wavelength $\lambda = (2\pi/mT)^{1/2}$. Therefore, the virial approach makes model-independent predictions for the conditions of the neutrinosphere, based only on the experimental scattering data.

In this Letter, we use the virial expansion to describe how neutrinos interact with low-density neutron matter. We focus on neutral-current interactions, and leave charged-current reactions and nuclear matter to future works. Our long-term goal is a reliable equation of state and consistent neutrino response for supernovae.

The free cross section per particle for neutrino–neutron elastic scattering is given by [8]

$$\frac{1}{N}\frac{d\sigma_0}{d\Omega} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left(C_a^2(3 - \cos\theta) + C_v^2(1 + \cos\theta)\right), \qquad (2)$$

* Corresponding author.
E-mail addresses: horowit@indiana.edu (C.J. Horowitz), schwenk@u.washington.edu (A. Schwenk).

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where G_F is the Fermi coupling constant, E_ν the neutrino energy, and θ the scattering angle. The weak axial coupling is $C_a = g_a/2$, with $g_a = 1.26$ the axial charge of the nucleon. The weak vector charge is $C_v = -1/2$ for scattering from a neutron. Eq. (2) neglects corrections of order E_ν/m from weak magnetism and other effects [9].

In the medium, this cross section is modified by the vector response $S_v(q)$ and the axial response $S_a(q)$

$$\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\big(g_a^2(3-\cos\theta)S_a(q) + (1+\cos\theta)S_v(q)\big), \quad (3)$$

where S_v and S_a describe the response of the system to density and spin fluctuations respectively, and $q = 2E_\nu \sin(\theta/2)$ denotes the momentum transfer. We will discuss the approximations for Eq. (3) in Section 2.3. In the following, we will use the virial expansion to provide model-independent results for the response in the long-wavelength ($q \to 0$) or forward-scattering limit.

This Letter is organized as follows. We extend the virial equation of state to spin-polarized matter in Section 2 and derive the consistent long-wavelength response. Further details on the virial equation of state can be found in Refs. [3,4]. In Section 3, we present results for the spin virial coefficients, the pressure and entropy of spin-polarized neutron matter, and the neutrino response. We compare our results to Brueckner calculations, and to random-phase approximation (RPA) response functions. Finally, we conclude in Section 4.

2. Formalism

The virial expansion is a general, model-independent approach for a dilute gas, provided the fugacity is small and for temperatures above any phase transitions. Under these conditions, the grand-canonical partition function can be expanded in powers of the fugacity. The second virial coefficient b_2 describes the z^2 term in this expansion and is directly related to the two-body scattering phase shifts [10,11]. The relation of the third virial coefficient to three-body scattering is not straightforward, and was only studied for special cases [12–14]. The virial expansion is not a perturbative $k_F a_s$ expansion, and its great advantage is that it includes bound states and scattering resonances on an equal footing.

2.1. Spin-polarized matter

The virial equation of state is easily generalized to spin-asymmetric systems. For two spin components, we denote the chemical potential for spin up and spin down particles by μ_+ and μ_-, with fugacity $z_+ = e^{\mu_+/T}$ and $z_- = e^{\mu_-/T}$ respectively. For the virial equation of state we expand the pressure in a power series of the fugacities

$$P = \frac{T}{\lambda^3}\big(z_+ + z_- + b_{n,1}\big(z_+^2 + z_-^2\big) + 2b_{n,0}z_+z_- + \mathcal{O}\big(z^3\big)\big). \quad (4)$$

The second virial coefficients $b_{n,1}$ for like spins and $b_{n,0}$ for opposite spins are related to the two-particle partition function and are given in terms of the scattering phase shifts in the next section. The densities follow from differentiating the pressure with respect to the fugacities. For the density of spin-up neutrons $n_+ = (\partial_{\mu_+} P)_T = z_+/T(\partial_{z_+} P)_T$ we thus have

$$n_+ = \frac{1}{\lambda^3}\big(z_+ + 2b_{n,1}z_+^2 + 2b_{n,0}z_+z_- + \mathcal{O}\big(z^3\big)\big), \quad (5)$$

and likewise for the density n_- of spin-down neutrons

$$n_- = \frac{1}{\lambda^3}\big(z_- + 2b_{n,1}z_-^2 + 2b_{n,0}z_-z_+ + \mathcal{O}\big(z^3\big)\big). \quad (6)$$

The total density n and the spin polarization Δ are then given by

$$n = n_+ + n_- \quad \text{and} \quad \Delta = \frac{n_+ - n_-}{n_+ + n_-}. \quad (7)$$

In this work, we truncate the virial expansion after second order in the fugacities. This leads to an equation of state that is thermodynamically consistent.

The dependence of the total density and the spin polarization on z_+ and z_- can be inverted to yield the virial equation of state directly in terms of $P(z_+(n,\Delta,T), z_-(n,\Delta,T),T)$. In practice, for a given spin polarization, we determine the spin-down fugacity as a function of the spin-up one $z_-(z_+,\Delta,T)$, and generate the virial equation of state in tabular form for a range of z_+ values. This maintains the thermodynamic consistency of the virial equation of state.

Finally, we will also discuss results for the entropy. The entropy density $s = S/V$ follows from differentiating the pressure with respect to the temperature $s = (\partial_T P)_{\mu_i}$. This leads to

$$s = \frac{5P}{2T} - n_+ \log z_+ - n_- \log z_- + \frac{T}{\lambda^3}\big(b'_{n,1}\big(z_+^2 + z_-^2\big) + 2b'_{n,0}z_+z_-\big), \quad (8)$$

where $b'(T) = db(T)/dT$ denotes the temperature derivative of the virial coefficients.

2.2. Spin virial coefficients

The second virial coefficient $b_{n,1}$ describes the interaction of two neutrons with the same spin projection. To this end, we generalize the second virial coefficient of the spin-symmetric system [4,10,11] to

$$b_{n,1}(T) = \frac{2^{1/2}}{\pi T}\int_0^\infty dE\, e^{-E/2T}\,\delta_1^{\text{tot}}(E) - 2^{-5/2}, \quad (9)$$

where $-2^{-5/2}$ is the free Fermi gas contribution and $\delta_1^{\text{tot}}(E)$ is the sum of the isospin and spin-triplet elastic scattering phase shifts at laboratory energy E. This sum is over all partial waves with angular momentum L and total angular momentum J allowed by spin statistics, and includes a degeneracy factor $(2J+1)/(2S+1)$