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$\bar{p}D$ atoms in realistic potentials

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Abstract

The $\bar{p}D$ atoms are studied in various realistic, popular $\bar{N}N$ potentials. The small energy shifts and decay widths of the atoms, which stem from the short-ranged strong interactions between the antiproton and deuteron, are evaluated in a well-established, accurate approach based on the Sturmian functions. The investigation reveals that none of the employed potentials, which reproduce the $\bar{N}N$ scattering data quite well, is able to reproduce the experimental data of the energy shifts of the $2p$ $\bar{p}D$ atomic states. The energy shifts of the $2p$ $\bar{p}D$ atomic states are very sensitive to the $\bar{N}N$ strong interactions, hence the investigation of the $\bar{p}D$ atoms is expected to provide a good platform for refining the $\bar{N}N$ interaction, especially at zero energy.

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1. Introduction

The second simplest antiprotonic atom is the antiprotonic deuteron atom $\bar{p}D$, consisting of an antiproton and a deuteron bound mainly by the Coulomb interaction but distorted by the short range strong interaction. The study of the $\bar{p}D$ atom is much later and less successful than for other exotic atoms like the protonium and pionium. Experiments were carried out at LEAR just in very recent years to study the properties of the $\bar{p}D$ atom [1,2]. Even prior to the experiments some theoretical works [3–5] had been carried out to study the $\bar{p}D$ atomic states in simplified $\bar{p}D$ interactions. Recently, a theoretical work [6] proposed a mechanism explaining the unexpected behavior, of the scattering lengths of $\bar{N}N$ and $\bar{p}D$ system, that the imaginary part of the scattering length does not increase with the size of the nucleus.

In the theoretical sector, one needs to overcome at least two difficulties in the study of the $\bar{p}D$ atom. First, the interaction between the antiproton and the deuteron core should be derived from realistic $\bar{N}N$ interactions, for example, the Paris $\bar{N}N$ potentials [7–9], the Dover–Richard $\bar{N}N$ potentials I (DR1) and II (DR2) [10,11], and the Kohno–Weise $\bar{N}N$ potential [12]. Even if a reliable $\bar{p}D$ interaction is in hands, the accurate evaluation of the energy shifts and decay widths (stemming for the strong $\bar{p}D$ interactions) and especially of the nuclear force distorted wave function of the atom is still a challenge. It should be pointed out that the methods employed in the works [3–5] are not accurate enough for evaluating the wave functions of the $\bar{p}D$ atoms.

In the present work we study the $\bar{p}D$ atom problem employing a properly adapted numerical method based on Sturmian functions [13]. The method accounts for both the strong *short* range nuclear potential (local and non-local) and the *long* range Coulomb force and provides directly the wave function of the $\bar{p}D$ system with complex eigenvalues $E = E_R - i\frac{\Gamma}{2}$. The protonium and pionium problems have been successfully investigated [14,15] in the numerical approach. The numerical method

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is much more powerful, accurate and much easier to use than all other methods applied to the exotic atom problem in history. The $\bar{p}D$ interactions in the work are derived from various realistic $\bar{N}N$ potential, which is state-dependent. The work is organized as follows. The $\bar{p}D$ interactions are expressed in Section 2 in terms of the $\bar{N}N$ interactions. In Section 3 the energy shifts and decay widths of the $1s$ and $2p$ $\bar{p}D$ atomic states are evaluated. Discussions and conclusions are given in Section 3, too.

2. $\bar{p}D$ interactions in terms of $\bar{N}N$ potentials

We start from the Schrödinger equation of the antiproton–deuteron system in coordinate space

$$\left(\frac{P_\rho^2}{2M_\rho}+\frac{P_\lambda^2}{2M_\lambda}+V_{12}(\vec{r}_2-\vec{r}_1)+V_{13}(\vec{r}_3-\vec{r}_1)+V_{23}(\vec{r}_3-\vec{r}_2)\right)\Psi(\vec{\lambda},\vec{\rho})=E\Psi(\vec{\lambda},\vec{\rho}) \tag{1}$$

where $\vec{\lambda}$ and $\vec{\rho}$ are the Jacobi coordinates of the system, defined as

$$\vec{\lambda}=\vec{r}_3-\frac{\vec{r}_1+\vec{r}_2}{2},\qquad \vec{\rho}=\vec{r}_2-\vec{r}_1, \tag{2}$$

$M_\rho=M/2$ and $M_\lambda=2M/3$ are the reduced masses. Here we have assigned, for simplicity, the proton and neutron the same mass M. Eq. (1) can be expressed in the form, where the strong interaction is expressed in the isospin basis,

$$\left(\frac{P_\rho^2}{2M_\rho}+\frac{P_\lambda^2}{2M_\lambda}+V_S+V_C\right)\Psi(\vec{\lambda},\vec{\rho})=E\Psi(\vec{\lambda},\vec{\rho}) \tag{3}$$

where V_S and V_C stand for the nuclear interaction and Coulomb force, respectively, and take the forms

$$V_S=V^0_{NN}(\vec{r}_2-\vec{r}_1)+\frac{1}{4}\big[V^0_{\bar{N}N}(\vec{r}_3-\vec{r}_1)+V^0_{\bar{N}N}(\vec{r}_3-\vec{r}_2)\big]+\frac{3}{4}\big[V^1_{\bar{N}N}(\vec{r}_3-\vec{r}_1)+V^1_{\bar{N}N}(\vec{r}_3-\vec{r}_2)\big], \tag{4}$$

$$V_C=\frac{1}{2}\big[V_C(\vec{r}_3-\vec{r}_1)+V_C(\vec{r}_3-\vec{r}_2)\big] \tag{5}$$

V^0 and V^1 in Eq. (4) are the isospin 0 and 1 nuclear interactions, respectively. Note that we have assigned $\vec{r}_{12}$ as the relative coordinate of the deuteron core.

One may express the interactions V_C and V_S in Eqs. (4) and (5) in terms of the interactions of certain $\bar{N}N$ states. In the $|JMLS\rangle$ basis of the $\bar{p}D$ states

$$|JMLS\rangle=\big|(L_\rho\otimes L_\lambda)_L\otimes(S_{12}\otimes S_3)_S\big\rangle_{JM} \tag{6}$$

we derive

$$\big(H_0+W_C(\lambda,\rho)+V^0_{NN}(\rho)+W_S(\lambda,\rho)\big)\Psi(\lambda,\rho)=E\Psi(\lambda,\rho) \tag{7}$$

with

$$H_0=\frac{P_\rho^2}{2M_\rho}+\frac{P_\lambda^2}{2M_\lambda} \tag{8}$$

W_C and W_S in Eq. (7) are respectively the Coulomb force and strong interaction between the antiproton and deuteron, and V^0_{NN} the interaction between the proton and neutron in the deuteron core. W_C and W_S are derived explicitly as

$$W_C(\lambda,\rho)=\frac{1}{2}\int_{-1}^{1}dx\,V_C(r_{13}), \tag{9}$$

$$W_S(\lambda,\rho)=\frac{1}{2}\int_{-1}^{1}dx\sum_{Q,Q'}\langle P|Q\rangle\langle Q|V_{\bar{N}N}(\vec{r}_{13})|Q'\rangle\langle Q'|P'\rangle \tag{10}$$

with

$$V_{\bar{N}N}(\vec{r}_{13})=\frac{1}{2}V^0_{\bar{N}N}(\vec{r}_{13})+\frac{3}{2}V^1_{\bar{N}N}(\vec{r}_{13}), \tag{11}$$

$$r_{13}\equiv|\vec{r}_1-\vec{r}_3|=\left(\lambda^2+\frac{\rho^2}{4}-\lambda\rho x\right)^{1/2} \tag{12}$$

where $x=\cos\theta$ with θ being the angle between $\vec{\lambda}$ and $\vec{\rho}$. In Eq. (10) $|P\rangle\equiv|JMLS\rangle$ and $|P'\rangle\equiv|JML'S\rangle$ are as defined in Eq. (6) while the states $|Q\rangle$ and $|Q'\rangle$ are

$$|Q\rangle=\big|(L_\sigma\otimes S_{13})_{J_\sigma}\otimes(L_\gamma\otimes S_2)_{J_\gamma}\big\rangle_{JM}, \tag{13}$$

$$|Q'\rangle=\big|(L'_\sigma\otimes S_{13})_{J_\sigma}\otimes(L_\gamma\otimes S_2)_{J_\gamma}\big\rangle_{JM}. \tag{14}$$

Here $\vec{\sigma}$ and $\vec{\gamma}$ are also the Jacobi coordinates of the system, defined as

$$\vec{\gamma}=\vec{r}_2-\frac{\vec{r}_1+\vec{r}_3}{2},\qquad \vec{\sigma}=\vec{r}_3-\vec{r}_1. \tag{15}$$

So defined the states $|Q\rangle$ and $|Q'\rangle$ is based on the consideration that the $\bar{N}N$ interactions can be easily expressed in the $|J_\sigma M_\sigma L_\sigma S_{13}\rangle$ basis of the $\bar{N}N$ states. Note that $\langle P|Q\rangle$ depends on not only the quantum numbers of the states $|P\rangle$ and $|Q\rangle$, but also λ, ρ and the angle θ between $\vec{\lambda}$ and $\vec{\rho}$ resulting from the projection of the orbital angular momenta between different Jacobi coordinates. We listed the integral kernels in Eq. (10), $\sum_{Q,Q'}\langle P|Q\rangle\langle Q|V(\vec{r}_{13})|Q'\rangle\langle Q'|P'\rangle$, for the lowest $\bar{p}D$ states in the approximation that the deuteron core is assumed in the S-state, as follows:

$$|P\rangle=|P'\rangle=|^2S_{1/2}\rangle\text{:}\quad \frac{3}{4}V_{\bar{N}N}(^1S_0)+\frac{1}{4}V_{\bar{N}N}(^3S_1),$$

$$|P\rangle=|P'\rangle=|^4S_{3/2}\rangle\text{:}\quad V_{\bar{N}N}(^3S_1),$$

$$|P\rangle=|P'\rangle=|^2P_{1/2}\rangle\text{:}\quad F_1^2\cdot\left[\frac{1}{12}V_{\bar{N}N}(^3P_0)+\frac{3}{4}V_{\bar{N}N}(^1P_1)+\frac{1}{6}V_{\bar{N}N}(^3P_1)\right],$$

$$|P\rangle=|P'\rangle=|^4P_{1/2}\rangle\text{:}\quad F_1^2\cdot\left[\frac{2}{3}V_{\bar{N}N}(^3P_0)+\frac{1}{3}V_{\bar{N}N}(^3P_1)\right],$$

$$|P\rangle=|P'\rangle=|^2P_{3/2}\rangle\text{:}\quad F_1^2\cdot\left[\frac{3}{4}V_{\bar{N}N}(^1P_1)+\frac{1}{24}V_{\bar{N}N}(^3P_1)+\frac{5}{24}V_{\bar{N}N}(^3P_2)\right],$$

$$|P\rangle=|P'\rangle=|^4P_{3/2}\rangle\text{:}\quad F_1^2\cdot\left[\frac{5}{6}V_{\bar{N}N}(^3P_1)+\frac{1}{6}V_{\bar{N}N}(^3P_2)\right],$$