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On vacuum structures of $N = 2$ LSUSY QED equivalent to $N = 2$ NLSUSY model

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Dedicated to the late Professor Julius Wess

Abstract

The vacuum structure of $N = 2$ linear supersymmetry (LSUSY) invariant QED, which is equivalent to $N = 2$ nonlinear supersymmetry (NLSUSY) model, is studied explicitly in two-dimensional space–time ($d = 2$). Two different isometries $SO(1,3)$ and $SO(3,1)$ appear for the vacuum field configuration corresponding to the various parameter regions. Two different field configurations of $SO(3,1)$ isometry describe the two different physical vacua, i.e. one breaks spontaneously both $U(1)$ and SUSY and the other breaks spontaneously SUSY alone.

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Supersymmetry (SUSY) in particle field theory [1,2] is a profound notions related to space–time symmetry. Therefore, the evidences of SUSY and its spontaneous breakdown [3–5] should be studied not only in (low energy) particle physics but also in cosmology, i.e. in a framework necessarily accommodating graviton.

In a series of works along these viewpoints, we have found group theoretically that the $SO(10)$ super-Poicaré (SP) group may be a unique and minimal group among all $SO(N)$ SP groups, which possesses a single irreducible linear (L) SUSY representation accommodating graviton and the standard model (SM) with just three generations of quarks and leptons [6].

The advocated difficulty for constructing non-trivial $N > 8$ SUSY (gravity) theory, the so-called no-go theorem based on S-matrix argument [7,8] can be circumvented by adopting the *nonlinear* (*NL*) *representation* of SUSY [9], i.e. the vacuum degeneracy of the fundamental action. Volkov–Akulov (VA) model [2] gives the NL representation of SUSY describing the dynamics of spin $1/2$ Nambu–Goldstone (NG) fermion accompanying the spontaneous SUSY breaking for $N = 1$.

The fundamental action (called nonlinear supersymmetric general relativity (NLSUSY GR)) of empty Einstein–Hilbert (EH) type for $N > 8$ SUSY (gravity) theory with $N > 8$ supercharges, i.e. the NLSUSY invariant interaction of N NG fermion with spin 2 graviton, has been constructed by extending the geometric arguments of Einstein general relativity (EGR) on Riemann space–time to a new space–time just inspired by NLSUSY, where tangent space–time is specified not only by x_a for $SO(1,3)$ but also by the Grassmanian ψ_α for isomorphic $SL(2C)$ of NLSUSY [10,11]. The compact isomorphic groups $SU(2)$ and $SO(3)$ for the gauge symmetry of 't Hooft–Polyakov monopole are generalized to the noncompact isomorphic groups $SO(1,3)$ and $SL(2C)$ for space–time symmetry and the consequent NLSUSY GR action possesses promising large symmetries isomorphic to $SO(10)$ SP [12,13]. These results mean that the no-go theorem is overcome (circumvented) in the sense that the non-trivial N-extended

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SUSY gravity theory with $N > 8$ has been constructed and graviton and N NG fermions with the spin difference $3/2$ can be coupled in a SUSY invariant way. We think that the geometric arguments of EGR principle has been generalized naturally, which accommodates *geometrically* spin $1/2$ matter as NG fermion accompanying spontaneous SUSY breaking encoded on tangent space–time as NLSUSY.

NLSUSY GR (called superon–graviton model (SGM) from composite viewpoint) on new empty space–time written in the form of the *vacuum* EH type is unstable due to NLSUSY structure of tangent space–time and decays (called *Big Decay* [13]) spontaneously into ordinary EH action with the cosmological constant Λ, NLSUSY action for N NG fermions (called *superons* as hypothetical spin $1/2$ objects) and their gravitational interactions on ordinary Riemann space–time, which ignites the Big Bang of the present universe. We have shown qualitatively that NLSUSY GR may potentially describe a new paradigm (SGM) for the SUSY unification of space–time and matter, where particular SUSY compositeness composed of superons for all (observed) particles except the graviton emerges as an ultimate feature of nature behind the familiar LSUSY models (MSSM, SUSY GUTs) [11,14] and SM as well. That is, all (observed) low energy particles may be eigenstates of $SO(N)$ SP expressed uniquely as the SUSY composites of N superons.

Due to the high nonlinearity of the SGM action we have not yet succeeded in extracting directly the evidence of such (low energy) physical meanings of SGM on curved Riemann space–time.

However, considering that SGM action reduces to the N-extended NLSUSY action in asymptotic Riemann-flat ($e^a{}_\mu \to \delta^a_\mu$) space–time after the Big Decay, it is interesting from the low energy physics viewpoint to construct the N-extended LSUSY theory equivalent to the N-extended NLSUSY model. The relation between $N = 1$ LSUSY representations and $N = 1$ NLSUSY representations in flat (Minkowski) space–time is well understood by using the superfield method [9,15]. The equivalence between $N = 1$ LSUSY *free* theory for LSUSY supermultiplet and $N = 1$ NLSUSY VA model for NG fermion is demonstrated by many authors [16–18] and $N = 2$ case as well [19], where each field of LSUSY supermultiplet is expressed uniquely as the composite of NG fermions of NLSUSY called SUSY invariant relations. Consequently we are tempted to imagine some composite structure (far) behind the SM and the familiar LSUSY models, e.g. MSSM and SUSY GUT.

Recently, we have shown explicitly by the heuristic arguments for simplicity in two space–time dimensions ($d = 2$) [20,21] that $N = 2$ LSUSY interacting QED is equivalent to $N = 2$ NLSUSY model. (Note that the minimal realistic SUSY QED in SGM composite scenario is described by $N = 2$ SUSY [19].)

In this Letter we study explicitly the vacuum structure of $N = 2$ LSUSY QED in the SGM scenario in $d = 2$ [21].

The $N = 2$ NLSUSY action for two superons (NG fermions) ψ^i ($i = 1, 2$) in $d = 2$ is written as follows,

$$\begin{aligned} S_{N=2\mathrm{NLSUSY}} &= -\frac{1}{2\kappa^2}\int d^2x\,|w| \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 + t^a{}_a + \frac{1}{2!}\left(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a\right)\right\} \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 - i\kappa^2\bar\psi^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4\left(\bar\psi^i \partial\!\!\!/ \psi^i \bar\psi^j \partial\!\!\!/ \psi^j - \bar\psi^i\gamma^a\partial_b\psi^i\bar\psi^j\gamma^b\partial_a\psi^j\right)\right\} \\ &= -\frac{1}{2\kappa^2}\int d^2x\left\{1 - i\kappa^2\bar\psi^i \partial\!\!\!/ \psi^i - \frac{1}{2}\kappa^4\epsilon^{ab}\left(\bar\psi^i\psi^j\partial_a\bar\psi^i\gamma_5\partial_b\psi^j + \bar\psi^i\gamma_5\psi^j\partial_a\bar\psi^i\partial_b\psi^j\right)\right\}, \end{aligned} \tag{1}$$

where $\kappa^2 = (\frac{c^4\Lambda}{8\pi G})^{-1}$ in the SGM scenario and

$$|w| = \det\left(w^a{}_b\right) = \det\left(\delta^a_b + t^a{}_b\right), \qquad t^a{}_b = -i\kappa^2\bar\psi^i\gamma^a\partial_b\psi^i. \tag{2}$$

While, the helicity states contained in $d = 2$ $N = 2$ LSUSY QED are the vector supermultiplet containing $U(1)$ gauge field

$$\begin{pmatrix} +1 \\ +\frac{1}{2},\ +\frac{1}{2} \\ 0 \end{pmatrix} + [\text{CPT conjugate}],$$

and the scalar supermultiplet for matter fields

$$\begin{pmatrix} +\frac{1}{2} \\ 0,\ 0 \\ -\frac{1}{2} \end{pmatrix} + [\text{CPT conjugate}].$$

The $N = 2$ LSUSY QED action in $d = 2$ for the massless case is written as follows [21],

$$\begin{aligned} S_{N=2\mathrm{SUSYQED}} = \int d^2x\Big[&-\frac{1}{4}(F_{ab})^2 + \frac{i}{2}\bar\lambda^i \partial\!\!\!/ \lambda^i + \frac{1}{2}(\partial_a A)^2 + \frac{1}{2}(\partial_a\phi)^2 + \frac{1}{2}D^2 - \frac{1}{\kappa}\xi D \\ &+ \frac{i}{2}\bar\chi \partial\!\!\!/ \chi + \frac{1}{2}\left(\partial_a B^i\right)^2 + \frac{i}{2}\bar\nu \partial\!\!\!/ \nu + \frac{1}{2}\left(F^i\right)^2 \end{aligned}$$