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Quantum collapse of a self-gravitating thin shell and statistical model of quantum black hole

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Abstract

The quantum collapse of a self-gravitating thin shell in the minisuperspace models is revisited on the assumption that the shell is composed of N distinguishable identical particles. The ground state of the shell is found and defined as a quantum black hole (QBH). We show that the energy of single particle in the QBH is dependent on N, and N has an up-limit for a stable QBH. The effective exciting energy of single particle is determined, which is universally $1/\sqrt{2}$ of the Planck energy for the full-filled QBHs. We also propose a simple statistical model of QBH and show that a QBH is full-filled at low temperatures and half-filled at high temperatures. The specific heat of QBH is found to be positive at low temperatures and the relation of the QBH mass with its temperature is obtained in the high-temperature limit of our model.
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The classical theory of gravity, general relativity, is suffered from the problems of singularities, where our present laws of physics break down [1]. It is believed that the singularities could be avoided when quantum effects are considered. A self-gravitating thin shell may be the simplest model of gravitational collapse, and it can be in a sense regarded as a touchstone for any quantum theories of gravity since they should provide at least a correct quantum collapse scenario for this simplest case. In this Letter, the quantum collapse of a self-gravitating thin shell is revisited and a statistical model of quantum black hole is proposed.

In general relativity, the collapse of a self-gravitating thin shell was first investigated by Israel [2,3]. The world sheet of the thin shell separates the space–time into two parts: the inside described by Minkowskian metric, and the outside by Schwarzschild metric. Einstein's field equations imply the equation of motion for the shell:

$$E = Mc^2\left[1+\left(\frac{1}{c}\frac{dR}{d\tau}\right)^2\right]^{1/2} - \frac{GM^2}{2R}, \tag{1}$$

where τ is the proper time along the shell of radius R and rest mass M, c is the speed of light, and G denotes the gravitational constant [3]. One can also choose the Minkowskian time in the flat space inside the shell as time variable, as first proposed by Kuchar [4], then Eq. (1) can be written in a suggestive form

$$E = Mc^2\left[1-\left(\frac{1}{c}\frac{dR}{dt}\right)^2\right]^{-1/2} - \frac{GM^2}{2R}, \tag{2}$$

where t denotes the time inside the shell. We note that Eq. (2) gives just the energy of a relativistic particle of rest mass M, which moves radially in a potential $-GM^2/2R$.

The quantum collapse of a self-gravitating thin shell was studied in the minisuperspace models by many authors [5–10]. Different choices of time lead to different quantum theories, which are not unitarily equivalent to each other [6,7,11]. Choosing the time along the shell as time variable, Berezin et al. proposed a theory with novel Hamiltonian and difference Schrödinger equation [5]. Choosing the time inside the shell as time variable, Hajicek, Kay and Kuchar (HKK) suggested another theory with a more natural and elegant form in [6], where the problem was reduced to the s-wave Klein–Gordon equation in a "Coulomb potential". The bound states obtained in the

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HKK model have the form of the Sommerfeld spectrum of a relativistic scalar hydrogen atom, which exist only if the rest mass of the shell is no more than one Planck mass.

In this Letter, we will choose the Minkowskian time in the flat space inside the shell as time variable following [6]. Such a choice is due to the heuristic relationship between self-gravitating thin shells and atoms implied by the HKK model as well as the consideration of mathematical simplicity. Another problem of minisuperspace models is that when most of the degrees of freedom are frozen, some property of the original system may be destroyed. For example, the thermodynamics of the system is lost [7,11]. In this Letter, we will revisit the HKK model on the assumption that the shell is composed of N identical particles.

Consider a thin shell composed of N identical particles with purely gravitational interaction and same radial motion. Using the time inside the shell as time variable, one can obtain from Eq. (2) the equations of motion for every particle:

$$E_i = mc^2\left[1-\left(\frac{1}{c}\frac{dr_i}{dt}\right)^2\right]^{-1/2} - \frac{NGm^2}{2r_i}, \tag{3}$$

where $E_i = E/N$, $m = M/N$, $r_i = R$ denote energy, rest mass and radial coordinate of the ith particle, respectively, and $i = 1, 2, \ldots, N$. The classical motion of the ith particle is generated by the Hamiltonian

$$H_i = \sqrt{(p_i c)^2 + \left(mc^2\right)^2} - \frac{NGm^2}{2r_i}, \tag{4}$$

where p_i denotes the radial momentum of the ith particle. The Hamiltonian for the shell is

$$H = \sum_{i=1}^{N} H_i = \sum_{i=1}^{N}\left[\sqrt{(p_i c)^2 + \left(mc^2\right)^2} - \frac{NGm^2}{2r_i}\right]. \tag{5}$$

Thus the shell can be regarded as a system of N non-interacting identical relativistic particles moving radially in a potential $-NGm^2/2r$, and the mathematical similarity between self-gravitating thin shells and relativistic scalar hydrogen atoms in the HKK model remains.

The quantum theory of the self-gravitating thin shell can be developed if one quantizes the classical system governed by Hamiltonian (5). For simplicity, here we assume that the identical particles are distinguishable and the interaction between them is purely gravitational, ignore the spins of the identical particles, and confine our attention to the bound states. From the quantum Hamiltonian corresponding to classical Hamiltonian (4), following [6], one can reduce the problem to the s-wave Klein–Gordon equation in a "Coulomb potential" and obtain the energy levels for bound s-states of single particle, which have the form of the Sommerfeld spectrum of a relativistic scalar hydrogen atom with $l = 0$ [12]:

$$\varepsilon_n(N) = mc^2\left[1 + \frac{N^2\alpha^2}{(n+\frac{1}{2}+[(\frac{1}{2})^2 - N^2\alpha^2]^{1/2})^2}\right]^{-1/2}. \tag{6}$$

Here $n = 0, 1, 2, \ldots$ and an effective fine structure constant is introduced by $\alpha = Gm^2/2\hbar c$. Eq. (6) with $n = 0$ yields the ground state energy of single particle:

$$\varepsilon_0(N) = mc^2\left[1 + \frac{N^2\alpha^2}{(\frac{1}{2}+[(\frac{1}{2})^2 - N^2\alpha^2]^{1/2})^2}\right]^{-1/2}. \tag{7}$$

Since the N distinguishable identical relativistic particles have same radial motion in the classical theory, they should occupy same single-particle state in the quantum theory. Therefore the shell in its ground state is composed of N identical particles in single-particle ground state. In this Letter, we briefly call the shell in its ground state "quantum black hole" (QBH), which may be regarded as the quantum correspondence to the remnants of classical gravitational collapse. Evidently the energy of a QBH is given by

$$E(N) = N\varepsilon_0(N). \tag{8}$$

There are two noticeable features of the QBH: the single-particle energy ε_0 is dependent on number of identical particles N, and ε_0 turns out to be complex for $N > 1/2\alpha$ which means a QBH will be unstable then. Firstly, we confine our attention to the case $N \leqslant 1/2\alpha$.

The maximum number of identical particles in a stable QBH can be defined by $N_{\max} \equiv 1/2\alpha$. If we parameterize the "filling factor" $N/N_{\max}$ by $N/N_{\max} = \sin\theta$, Eq. (7) can be greatly simplified as

$$\varepsilon_0(\theta) = mc^2\cos(\theta/2). \tag{9}$$

Since $N_{\max} \equiv 1/2\alpha \geqslant 1$ and $\alpha = Gm^2/2\hbar c$, we have $N_{\max} = (m_P/m)^2$ and $m \leqslant \sqrt{\hbar c/G} = m_P$, which suggest that the large $N_{\max}$ corresponds to the small m and a QBH can be formed only by identical particles of rest mass no more than one Planck mass. In the following discussion we assume $m \ll m_P$, thus the parameter θ can be treated as a continuous variable. As an application of Eq. (9), one can obtain that for a full-filled QBH ($\theta = \pi/2$), $\varepsilon_0 = mc^2/\sqrt{2}$, and for a half-filled one ($\theta = \pi/6$), $\varepsilon_0 = \cos(\pi/12)mc^2$.

The mass of a QBH can be defined by

$$M = E/c^2 = \frac{m_P^2}{m}\sin\theta\cos(\theta/2). \tag{10}$$

Evidently M is inversely proportional to m for given filling factor. One can find that M reaches its maximum at $\sin\theta = N/N_{\max} = 2\sqrt{2}/3$ for given small m. The result is universal for $m \ll m_P$. The case of large m is also interesting. For example, $M = m_P/\sqrt{2}$ for ($m = m_P$, $N = N_{\max} = 1$), which gives the smallest mass of a full-filled QBH, and $M = m_P$ for ($m = m_P/\sqrt{2}$, $N = N_{\max} = 2$), corresponding to a full-filled QBH with just one Planck mass. One can also find that M reaches its maximum at full filling for $N_{\max} \leqslant 13$.

Following the same analysis in the HKK model [6] one can estimate the size of a QBH from the wave function of single-particle ground state $\psi_0(\vec{r})$. For $\theta \to 0$, the wave function is similar to that of a non-relativistic hydrogen atom and the size of the QBH is simply given by "Bohr radius", $r_0 = \lambdabar_C/N\alpha$. Here $\lambdabar_C = \hbar/mc$, denotes the Compton wavelength of the identical particle. It is obvious that r_0 is much larger than r_S, Schwarzschild radius of the QBH in this case. As θ increases,

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