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A comment on the topological phase for anti-particles in a Lorentz-violating environment

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Abstract

Recently, a scheme to analyse topological phases in quantum mechanics by means of the non-relativistic limit of fermions non-minimally coupled to a Lorentz-breaking background has been proposed. In this Letter, we show that the fixed background, responsible for the Lorentz-symmetry violation, may induce opposite Aharonov–Casher phases for a particle and its corresponding anti-particle. We then argue that such a difference may be used to investigate the asymmetry for particle/anti-particle as well as to propose bounds on the associated Lorentz-symmetry violating parameters.

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The Standard Model of particle physics is based on Lorentz- and CPT-invariances as fundamental symmetries that have been confirmed in numerous experiments [1–5]. Actually, the invariance under the combined CPT-transformation is a consequence of first principles of relativistic quantum field theory. The most immediate consequence of CPT symmetry is the equality of mass and lifetime for a particle and its corresponding anti-particle. The best tests in this direction come from the limits on the mass difference between K^0 and $\overline{K^0}$ [3], high precision measurements of the anomalous magnetic moments of the electrons, positrons and mesons (confined in a Penning trap) [4], and clock-comparison experiments [5].

Lorentz-violating theories are presently studied as a possible extension of the Standard Model of particle physics. This proposal has been pushed forward by Colladay and Kosteletcký [6], who devised a Standard Model extension (SME) incorporating all tensor terms stemming from the spontaneous symmetry breaking of a more fundamental theory. In this case, an effective action breaks Lorentz symmetry at the particle frame, but keeps unaffected the $SU(3)\times SU(2)\times U(1)$ gauge structure of the underlying fundamental theory. This fact is of relevance in that it indicates that the effective model may preserve some good properties of the original theory, like causality, unitarity and stability.

In the context of gauge theories endowed with Lorentz violation, some efforts have been recently devoted to investigate interesting features of relativistic quantum-mechanical model involving the presence of fermions. Indeed, considering the Dirac equation enriched by of a sort of non-minimal coupling, significant consequences on the particle behavior has been observed, as pointed out in the works of Ref. [7]. In these papers,

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the analysis of the non-relativistic regime of the Dirac equation has revealed that topological quantum phases are induced whenever the fermion field is coupled to the fixed background and the gauge field in a non-minimal way. More specifically, it has been found out that a neutral particle acquires a magnetic moment (induced by the background), which originates the Aharonov–Casher (AC) phase subject to the action of an external electric field [10]. It is worth stressing that the standard Aharonov–Casher phase is interpreted as a Lorentz change in the observer frame. In our proposal, it rather emerges as a phase whose origin is ascribed to the presence of a privileged direction in the space–time, set up by the fixed background. Since in this kind of model Lorentz invariance in the particle frame is broken, the Aharonov–Casher effect could not here be obtained by a suitable Lorentz change in the observer frame.

In this Letter, rather than focusing the quest for CPT-violation on possible particle/anti-particle mass or lifetime differences, we elect the Aharonov–Casher effect as an effective probe to detect particle/anti-particle asymmetry in a Lorentz violating environment. Based on the result of Ref. [7], we show that particles and anti-particles, even if spinless and neutral, may pick up opposite AC-phases in their wave functions. This may lead to interference effects that could be used to verify potential particle/anti-particle asymmetries as a consequence of Lorentz and/or CPT-violation induced by some background. This outcome may be also used to set tight bounds on the CPT-violating parameters. Our Letter is outlined as follows. First, we begin by reviewing the standard minimal coupling of a charged anti-particle, which gives the Aharonov–Bohm (AB) phase. We then observe that for the non-relativistic (Pauli) Hamiltonian, the AB phase for a particle and its anti-particle presents no modification. Afterwards, we contemplate the standard non-minimal coupling (the Pauli interaction) that gives rise to the Aharonov–Casher phase for a non-relativistic neutral particle endowed with magnetic dipole moment, showing that in this case the AC phase does not undergo any change under CPT transformation as well. Finally, we investigate the case of main interest: some possible Lorentz-violating non-minimal couplings that induce AC phases. In the situations considered here, it is demonstrated that the AC phases for a particle and its anti-particle exhibit opposite signs.

Our starting point is an investigation on the Aharonov–Bohm phase induced for the anti-particle by the usual minimal coupling to the electromagnetic field, which respects the CPT-symmetry. We then write the gauge invariant Dirac equation for the anti-particle, which may be obtained from the standard Dirac equation after a CPT transformation. The spinor wave function transforms under C, P and T operations as follows: $\Psi \xrightarrow{\mathcal{P}} \eta_P \gamma^0 \Psi$, $\Psi \xrightarrow{\mathcal{C}} \eta_C C \bar{\Psi}^t$ and $\Psi \xrightarrow{\mathcal{T}} \eta_T \gamma^1 \gamma^3 \Psi^*$, where the η's correspond to the phases associated with each transformation and C is the charge conjugation matrix, given in this representation[1] by $C = i\gamma^0\gamma^2$. Considering the overall effect of these transformations, the spinor function describing the anti-particle changes as below

$$\Psi_{CPT} = \gamma_5 \Psi. \tag{1}$$

Moreover, under CPT-transformation[2] it holds: $\partial_\mu \overset{CPT}{\rightarrow} -\partial_\mu$, $A_\mu \overset{CPT}{\rightarrow} -A_\mu$, $F_{\mu\nu} \overset{CPT}{\rightarrow} F_{\mu\nu}$. So, for the anti-particle, the Dirac equation reads as

$$\left(i\gamma^\mu \partial_\mu + m - e\gamma^\mu A_\mu\right)\gamma_5 \Psi = 0. \tag{2}$$

The first case to be discussed here concerns the AB phase; we take both particle and anti-particle are minimally coupled to the electromagnetic vector potential via the covariant derivative with minimal coupling. To work out the non-relativistic limit of the Dirac equation for the anti-particle, the spinor Ψ should be written in terms of the so-called small and large components, as it is usually done. Thereby, there appear two coupled equations for the 2-component spinors, that, once decoupled, yield the non-relativistic Hamiltonian

$$H = -\frac{1}{2m}\vec{\Pi}^2 - e\varphi + \frac{e}{2m}\vec{\sigma} \cdot (\vec{\nabla} \times \vec{A}), \tag{3}$$

and the canonical conjugated momentum for the strong component, $\vec{\Pi} = (\vec{p} - e\vec{A})$. This puts in evidence that the energy of the anti-particle (before reinterpretation) has opposite sign in comparison with the energy for the corresponding particle, as a consequence of CPT transformation. However, the phase factor associated with the Aharonov–Bohm effect does not change its sign. Indeed, the bilinear term $(A^\mu J_\mu)$ of the Lagrangian, where the AB phase stems from, remains invariant.

The Aharonov–Casher phase for the anti-particle can be analysed on the same grounds. To work it out, we consider an electrically neutral particle (not necessarily a Majorana fermion), described by a spinor, Ψ, whose behavior is governed by the Dirac equation non-minimally coupled to gauge field

$$\left(i\gamma^\mu \partial_\mu - m + \frac{1}{2}\mu F^{\mu\nu}\Sigma_{\mu\nu}\right)\Psi = 0, \tag{4}$$

where the non-minimal term under CPT transformation goes as: $\mu F^{\mu\nu}\Sigma_{\mu\nu} \overset{CPT}{\rightarrow} \mu F^{\mu\nu}\Sigma_{\mu\nu}$. To isolate the AC-phase for the anti-particle, the magnetic field is set to zero, $F^{ij} = 0$. After simple algebraic manipulations, $F^{\mu\nu}\Sigma_{\mu\nu}$ turns into $i\mu E^i \gamma_i \gamma_0$, so that the Dirac equation takes on the form: $(-i\gamma^\mu \partial_\mu - m - i\mu E^i \gamma^i \gamma^0)\gamma_5 \Psi = 0$. Again, to compute the non-relativistic limit, the spinor Ψ is split into strong and weak components, yielding the following Hamiltonian:

$$H = \frac{1}{2m}(\vec{p} - \vec{E} \times \vec{\mu})^2 - \frac{\mu^2 \vec{E}^2}{2m}, \tag{5}$$

while $\vec{\Pi} = (\vec{p} - \vec{E} \times \vec{\mu})$ is the conjugate momentum. This shows that, as in the case of the AB phase, the phase acquired by the anti-particle is not modified. This phase invariance is obviously a consequence of the Lorentz and CPT symmetries of the non-minimal coupling above.

We now go on to investigate the effect of a unusual non-minimal coupling on the topological AC phase. As it has been

[1] Here, we adopt the usual Dirac representation for the γ-matrices.

[2] We adopt the prescriptions found in Ref. [11].