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Cosmological dynamics with modified induced gravity on the normal DGP branch

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ABSTRACT

In this Letter we investigate cosmological dynamics on the normal branch of a DGP-inspired scenario within a phase space approach where induced gravity is modified in the spirit of $f(R)$-theories. We apply the dynamical system analysis to achieve the stable solutions of the scenario in the normal DGP branch. Firstly, we consider a general form of the modified induced gravity and we show that there is a standard de Sitter point in phase space of the model. Then we prove that this point is stable attractor only for those $f(R)$ functions that account for late-time cosmic speed-up.

1. Introduction

There are many astronomical evidences supporting the idea that our universe is currently undergoing a speed-up expansion [1]. Several approaches are proposed in order to explain the origin of this novel phenomenon. These approaches can be classified in two main categories: models based on the notion of *dark energy* which modify the matter sector of the gravitational field equations and those models that modify the geometric part of the field equations generally dubbed as *dark geometry* in literature [2,3]. From a relatively different viewpoint (but in the spirit of dark geometry proposal), the braneworld model proposed by Dvali, Gabadadze and Porrati (DGP) [4] explains the late-time cosmic speed-up phase in its self-accelerating branch without recourse to dark energy [5]. However, existence of ghost instabilities in this branch of the solutions makes its unfavorable in some senses [6]. Fortunately, it has been revealed recently that the normal, ghost-free DGP branch has the potential to explain late-time cosmic speed-up if we incorporate possible modification of the induced gravity in the spirit of $f(R)$-theories [7]. This extension can be considered as a manifestation of the scalar-tensor gravity on the brane. Some features of this extension are studied recently [8,9].

Within this streamline, in this Letter we study the phase space of the normal DGP cosmology where induced gravity is modified in the spirit of $f(R)$-theories. We apply the dynamical system analysis to achieve the stable solutions of the model. To achieve this goal, we firstly consider a general form of the modified induced gravity. We obtain fixed points via an autonomous dynamical system where the stability of these points depends explicitly on the form of the $f(R)$ function. There are also de Sitter phases, one of which is a stable phase explaining the late-time cosmic speed-up. Secondly, in order to determine the stability of critical points and for the sake of clarification, we specify the form of $f(R)$ by adopting some cosmologically viable models. The phase spaces of these models are analyzed fully and the stability of critical points are studied with details.

2. DGP-inspired $f(R)$ gravity

2.1. The basic equations

Modified gravity in the form of $f(R)$-theories are derived by generalization of the Einstein–Hilbert action so that R (the Ricci scalar) is replaced by a generic function $f(R)$ in the action

$$S=\int d^4x\sqrt{-g}\left(\frac{f(R)}{2\kappa^2}+\mathbf{L}_m\right), \tag{1}$$

where $\mathbf{L}_m$ is the matter Lagrangian and $\kappa^2=8\pi G$. Varying this action with respect to the metric gives

$$G_{\mu\nu}=\kappa^2T^{(tot)}_{\mu\nu}=\kappa^2\left(T^{(m)}_{\mu\nu}+T^{(f)}_{\mu\nu}\right)=\kappa^2\frac{\widetilde{T}^{(m)}_{\mu\nu}+\widetilde{T}^{(f)}_{\mu\nu}}{f'} \tag{2}$$

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where $\widetilde{T}^{(m)}_{\mu\nu} = diag(\rho, -p, -p, -p)$ is the stress-energy tensor for standard matter, which is assumed to be a perfect fluid and by definition $f' \equiv \frac{df}{dR}$. Also $\widetilde{T}^{(f)}_{\mu\nu}$ is the stress-energy tensor of the *curvature fluid* that is defined as follows

$$\widetilde{T}^{(f)}_{\mu\nu} = \frac{1}{2} g_{\mu\nu}\big[f(R) - Rf'\big] + f'^{;\alpha\beta}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}). \tag{3}$$

By substituting a flat FRW metric into the field equations, one achieves the analogue of the Friedmann equations as follows [10]

$$3f'H^2 = \kappa^2\rho_m + \left[\frac{1}{2}\big(f(R) - Rf'\big) - 3H\dot{f}'\right], \tag{4}$$

$$-2f'\dot{H} = \kappa^2\rho_m + \dot{R}^2 f''' + (\ddot{R} - H\dot{R})f'', \tag{5}$$

where a dot marks the differentiation with respect to the cosmic time. In the next step, following [9] we suppose that the induced gravity on the DGP brane is modified in the spirit of $f(R)$ gravity. The action of this DGP-inspired $f(R)$ gravity is given by

$$S = \frac{1}{2\kappa_5^3}\int d^5x\sqrt{-g}\mathcal{R} + \int d^4x\sqrt{-q}\left(\frac{f(R)}{2\kappa^2} + \mathbf{L}_m\right), \tag{6}$$

where g_{AB} is the five dimensional bulk metric with Ricci scalar $\mathcal{R}$, while q_{ab} is induced metric on the brane with induced Ricci scalar R. The Friedmann equation in the *normal branch* of this scenario is written as [9]

$$3f'H^2 = \kappa^2\big(\rho_m + \rho^{(f)}\big) - \frac{3H}{r_c}, \tag{7}$$

where $r_c = \frac{G^{(5)}}{G^{(4)}} = \frac{\kappa_5^2}{2\kappa^2}$ is the DGP crossover scale with dimension of [*length*] and marks the IR (infra-red) behavior of the DGP model. The Raychaudhuri equation is written as follows

$$\dot{H}\left(1 + \frac{1}{2Hr_c f'}\right) = -\frac{\kappa^2\rho_m}{2f'} - \frac{\dot{R}^2 f''' + (\ddot{R} - H\dot{R})f''}{2f'}. \tag{8}$$

To achieve this equation we have used the continuity equation for $\rho^{(f)}$ as

$$\dot{\rho}^{(f)} + 3H\left(\rho^{(f)} + p^{(f)} + \frac{\dot{R}f''}{r_c(f')^2}\right) = \frac{\kappa^2\rho_m\dot{R}f''}{(f')^2}, \tag{9}$$

where the energy density and pressure of the *curvature fluid* are defined as follows

$$\rho^{(f)} = \frac{1}{\kappa^2}\left(\frac{1}{2}\big[f(R) - Rf'\big] - 3H\dot{f}'\right), \tag{10}$$

$$p^{(f)} = \frac{1}{\kappa^2}\left(2H\dot{f}' + \ddot{f}' - \frac{1}{2}\big[f(R) - Rf'\big]\right). \tag{11}$$

After presentation of the required field equations, we analyze the phase space of the model fully to explore cosmological dynamics of this setup.

2.2. A dynamical system viewpoint

The dynamical system approach is a convenient tool to describe dynamics of cosmological models in phase space. In this way, we rewrite Eq. (7) in a dimensionless form as

$$1 = \frac{\rho_m}{3H^2 f'} - \frac{1}{Hr_c f'} + \frac{f(R)}{6H^2 f'} - \frac{R}{6H^2} - \frac{\dot{f}'}{Hf'}. \tag{12}$$

In the present study, we firstly consider a generic form of the $f(R)$ function, so that one can define the dynamical variables independent of the specific form of the $f(R)$ function as follows (see for instance Ref. [10])

$$x_1 = \frac{\rho_m}{3H^2 f'}, \qquad x_2 = -\frac{1}{Hr_c f'}, \qquad x_3 = \frac{f}{6H^2 f'},$$
$$x_4 = -\frac{R}{6H^2}, \qquad x_5 = -\frac{\dot{f}'}{Hf'}. \tag{13}$$

Also we define the following quantities

$$m \equiv \frac{d\ln f'}{d\ln R} = \frac{Rf''}{f'}, \tag{14}$$

$$r \equiv -\frac{d\ln f}{d\ln R} = -\frac{Rf'}{f} = \frac{x_4}{x_3}. \tag{15}$$

We note that a constant value of m leads to the models with $f(R) = \xi_1 + \xi_2 R^{1+m}$ where the parameter m shows the deviation of the background dynamics from the standard model and ξ_1 and ξ_2 are constants. However, in general the parameter m depends on R and R itself can be expressed in terms of the ratio $r = \frac{x_4}{x_3}$. This means that m is a function of r, that is, $m = m(r)$. Based on the new variables, the Friedmann equation becomes a constraint equation so that we can express one of these variables in terms of the others. Introducing a new time variable $\tau = \ln a = N$ and eliminating x_1 (by using the Friedmann constraint equation) we obtain the following autonomous system

$$\frac{dx_2}{dN} = x_2(x_5 + x_4 + 2), \tag{16}$$

$$\frac{dx_3}{dN} = -\frac{x_4 x_5}{m} + x_3(2x_4 + x_5 + 4), \tag{17}$$

$$\frac{dx_4}{dN} = \frac{x_4 x_5}{m} + x_4(2x_4 + 4), \tag{18}$$

$$\frac{dx_5}{dN} = (x_2 + x_5)(x_5 + x_4) + 1 - 3x_3 - 5x_4 - 2x_2, \tag{19}$$

and

$$x_1 \equiv \Omega_m = 1 - x_2 - x_3 - x_4 - x_5. \tag{20}$$

The deceleration parameter which is defined as $q = -1 - \frac{\dot{H}}{H^2}$, now can be expressed as

$$q = 1 + x_4, \tag{21}$$

and the effective equation of state parameter of the system is defined by

$$\omega_{eff} = -1 - \frac{2\dot{H}}{3H^2}. \tag{22}$$

2.3. Critical points and their stability

The critical points of the scenario and some of their properties are listed in Table 1. In this table, Γ is defined as

$$\Gamma \equiv \frac{1}{2}\frac{4m^2 - 9m + 2 \pm \sqrt{-160m^4 + 272m^3 - 111m^2 + 4m + 4}}{2m^2 - 3m + 1}.$$

We consider only the plus sign of this equation in our forthcoming arguments. The minus sign does not create suitable cosmological behavior since it leads to $w_{eff} < -10$ or $w_{eff} > 0.7$ for point $\mathcal{E}$.

In Table 1, the critical points $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are independent of the form of $f(R)$. Nevertheless, the stability of these points depends on the form of $f(R)$ explicitly. The critical curve $\mathcal{D}$ exists just for $f(R)$ models with $m(r = -\frac{1}{2}) = \frac{1}{2}$ (for instance, in models of the form $f(R) = R + \gamma R^{-n}$ that m is defined as $m(r) = -\frac{n(1+r)}{r}$, the critical curve $\mathcal{D}$ exists just for $n = \frac{1}{2}$). The value of the effective